

# Scattering matrix elements

Scattering matrices contain all polarizing properties of the samples of randomly oriented particles and play an important role in radiative transfer processes. If the incident light is unpolarized only a few elements of the scattering matrix (the first column) suffice to fix the flux and state of polarization of the light scattered once by the sample. But the complete scattering matrix is indispensable for accurate multiple scattering calculations, since unpolarized light becomes polarized after being scattered.

For the definition of the scattering matrix we make use of the fact that the flux and polarization of a quasi-monochromatic beam of light can be represented by a column vector  $\pi\Phi = \pi\{\Phi_1, \Phi_2, \Phi_3, \Phi_4\}$ , which is called a flux vector [1; 2; 3].

If light is scattered by a sample of randomly oriented particles and time reciprocity applies, as is the case in our experiment, the flux vectors of the incident beam and the scattered beam are related by a  $4 \times 4$  scattering matrix, for each scattering angle  $\theta$ , as follows [1, Section 5.22] (see also normalization of the scattering matrix).

$$\Phi = \frac{\lambda^2}{4\pi^2 D^2} \begin{pmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{12} & F_{22} & F_{23} & F_{24} \\ -F_{13} & -F_{23} & F_{33} & F_{34} \\ F_{14} & F_{24} & -F_{34} & F_{44} \end{pmatrix} \Phi_0, \quad (1)$$

where  $\pi\Phi_0$  and  $\pi\Phi$  are the flux vectors of the incident and scattered beams, respectively,  $\lambda$  is the wavelength, and  $D$  is the distance from the particles to the detector. The matrix,  $\mathbf{F}$ , with elements  $F_{ij}$  is called the scattering matrix of the sample and refers to light that has been scattered once. Its elements depend on the scattering angle, but not on the azimuthal angle. Here the plane of reference is the scattering plane, i.e., the plane containing the incident and the scattered light. It follows from Eq. (1) that there are in general 10 different matrix elements.

For unpolarized incident light,  $F_{11}(\theta)$  is proportional to the flux of the scattered light and is also called scattering function or phase function. The ratio  $-F_{12}(\theta)/F_{11}(\theta)$  equals the degree of linear polarization of the scattered light if the incident light is unpolarized and  $F_{13}(\theta) = 0$ . Note further that we must have  $|F_{ij}(\theta)/F_{11}(\theta)| \leq 1$  [4].

In the data base, all elements, except  $F_{11}(\theta)$ , are given relative to  $F_{11}(\theta)$ , i.e., we list  $-F_{12}(\theta)/F_{11}(\theta)$ ,  $F_{22}(\theta)/F_{11}(\theta)$ ,  $F_{34}(\theta)/F_{11}(\theta)$ ,  $F_{33}(\theta)/F_{11}(\theta)$ , and  $F_{44}(\theta)/F_{11}(\theta)$ .

For aerosol particles, all measured values of  $F_{11}(\theta)$  are normalized so that they equal one for  $\theta = 30^\circ$ . See the link [normalization of the scattering matrix](#) for more information about this subject. In addition to each measured matrix element (ratio) value, the experimental (1-sigma) error is given. We refrained from listing the four element ratios  $F_{13}(\theta)/F_{11}(\theta)$ ,  $F_{14}(\theta)/F_{11}(\theta)$ ,  $F_{23}(\theta)/F_{11}(\theta)$ , and  $F_{24}(\theta)/F_{11}(\theta)$ , since, although we measured these ratios for most aerosol particles in the database, these measured values never showed deviations from zero by more than the experimental errors (see also [5]). This is consistent with scattering samples consisting of randomly oriented particles with equal amounts of particles and their mirror particles [1].

The scattering matrices given in the database satisfy the Cloude (coherency matrix) test [6] within the accuracy of the measurements [7; 8; 9].

Different conventions are occasionally used for Stokes parameters and, consequently, for the sign of the matrix element  $F_{34}(\theta)$ . The convention employed here is in accordance with [1] and [2].

## References

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