[The connection between missing stellar cusps in galactic nuclei and general relativity]

Extreme mass ratio inspirals and stellar distribution around MBHs

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Outline

I. EMRIs : Why and for which nuclei?

II. Actual data – A particular subject of interest: Our own galactic center

III. Distribution of stars around MBHs

IIII. A missing cusp

V. Self-consistent calculation of EMRI events

VI. The role of SMS : Boosting EMRI event rate (independently of what it is)

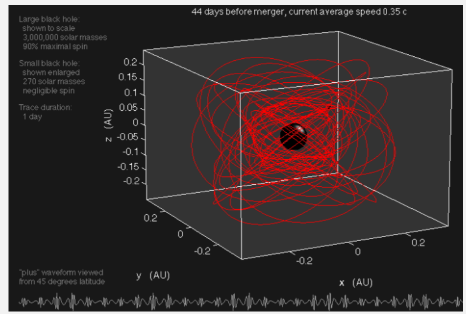
VII. Outlook and conclusions

Based on:

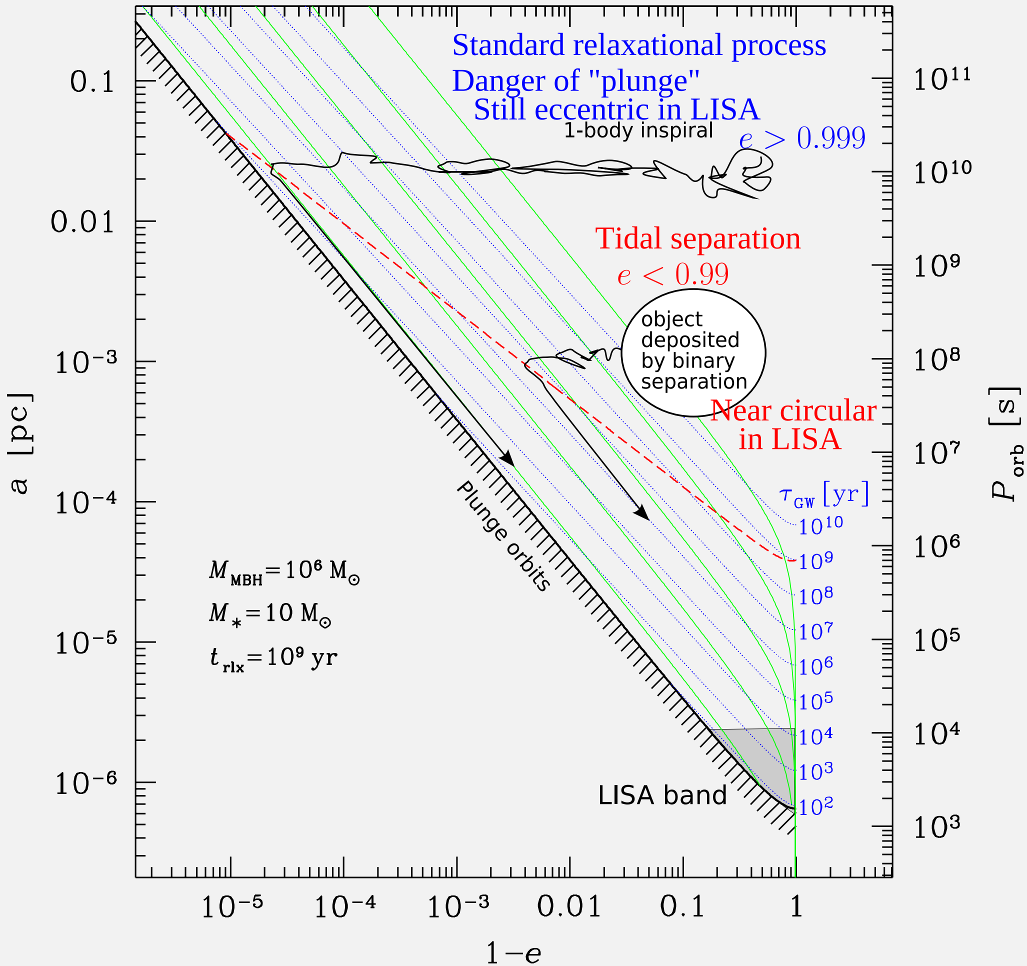
Amaro-Seoane, Living Review Relativity 2011 this spring
Preto & Amaro-Seoane 2010 ApJ Letts
Amaro-Seoane & Preto 2011 CQG
Amaro-Seoane, Preto & Freitag, to be submitted 2011

EMRIs

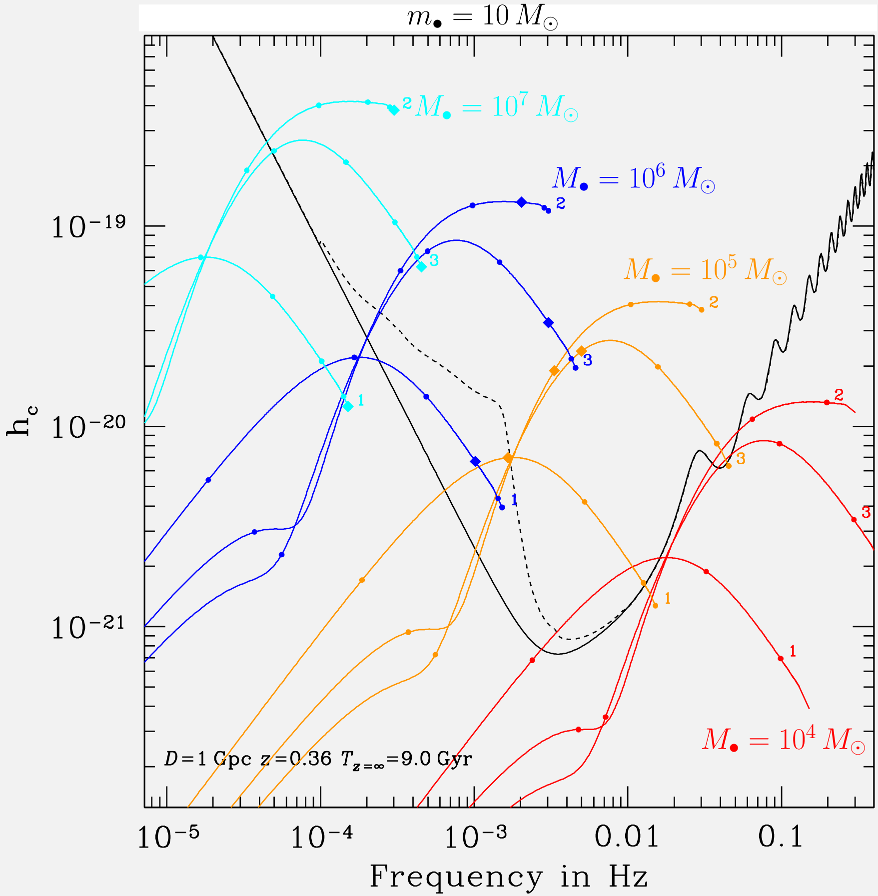
- Stellar mass object spiraling into $10^4 - 10^7\, M_\bullet$
- This range of masses corresponds to relaxed nuclei (!!!)
- Only compact objects (extended stars disrupted early)
- Stellar BH detectable to 3 Gpc ($z \sim 1$, local universe)
- EMRIs will allow "geo"desic mapping of space-time
- Tests of alternative theories (e.g. Chern-Simons gravity)
- Establishes MBH existence; measures mass and spin with unprecedent precission
- Main astrophysical scenario predict eccentric orbits in LISA band :
  Rich (but/thus complex and difficult) signal



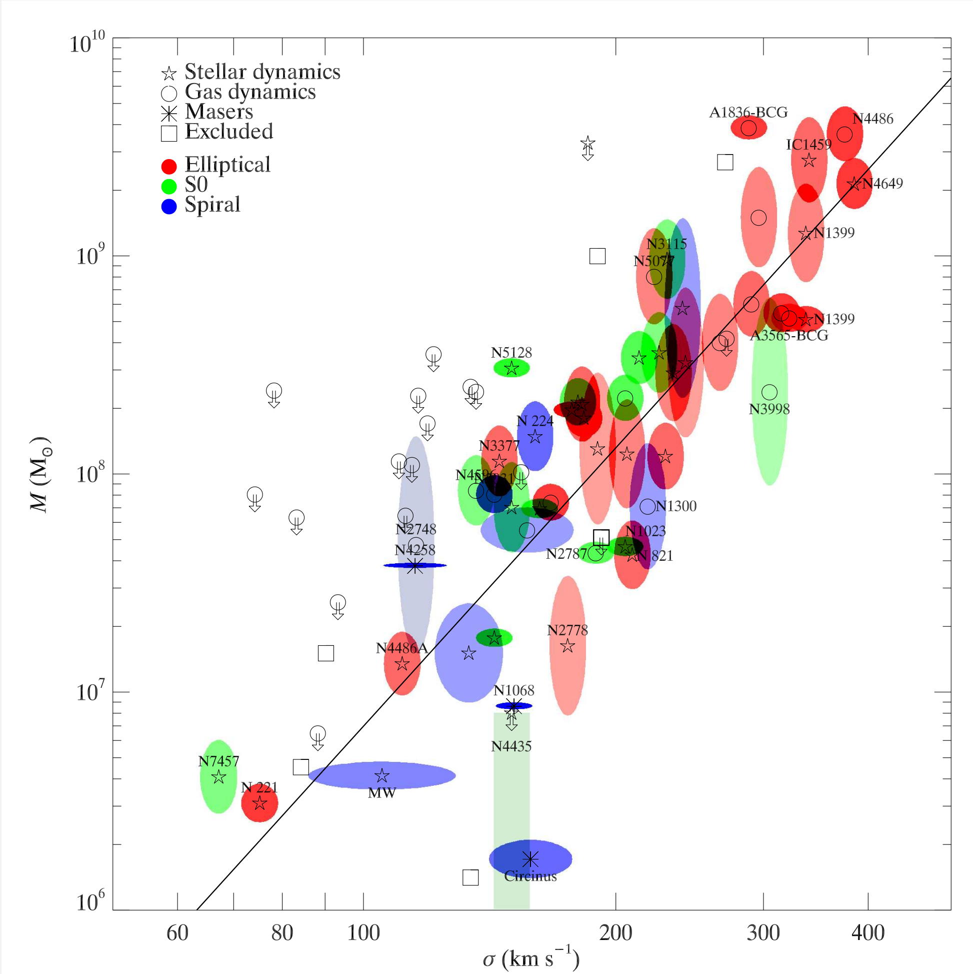
Movie by Steve Drasco, AEI; frequencies have been raised 19 octaves to make them audible



(Amaro-Seoane et al 2007) See extra-material!



$m_\bullet = 10\,M_\odot$
$M_\bullet = 10^7\,M_\odot$
$M_\bullet = 10^6\,M_\odot$
$M_\bullet = 10^5\,M_\odot$
$M_\bullet = 10^4\,M_\odot$
h_c
10^{-19}
10^{-20}
10^{-21}
10^{-5} 10^{-4} 10^{-3} 0.01 0.1
Frequency in Hz
D=1 Gpc z=0.36 $T_{z=\infty}$=9.0 Gyr



(Gülketin et al 2009)

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EMRIs
Distribution of stars
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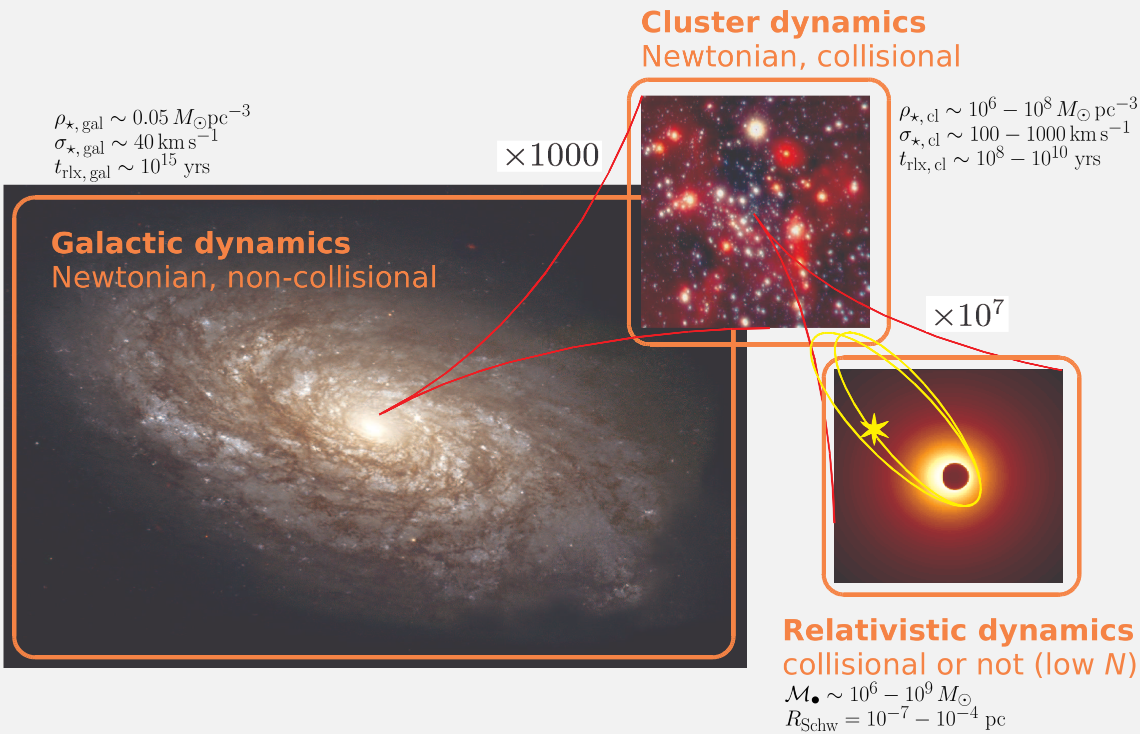
Analytical rates
Close

So... How many of these do you get a year?



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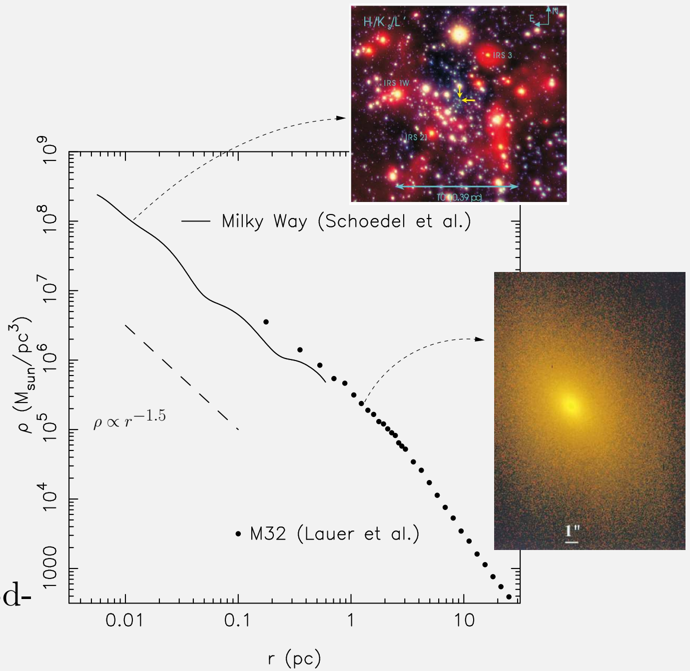
- ⋆ 0th question: How many stars? How do they distribute?
- ⋆ **Very few observations**
- ⋆ R_h difficult to resolve
- ⋆ To study inner region have to assume underlying pop
- ⋆ Deproject observation
- ⋆ Assume observed star is tracing invisible pop
- ⋆ Considerable amount of modelling
- ⋆ MW and M32 similar profile
- ⋆ Coincidence?



(Adapted from Merritt 2006)

Distribution of stars

- Classical problem in stellar dynamics (**38 years!**)

> Only a fool tries to solve a complicated problem when he does not even understand the simplest idealization – Donald Lynden-Bell

- EMRI rates depend $\rho_{\rm CO}$ and physics …within $a_{\rm GW} \sim \mathcal{O}(0.01\,{\rm pc})$ (Hopman & Alexander 2005, Amaro-Seoane et al 2007, 2011)
- Statistical thermal equilibrium $f(E) \propto e^{-E/\sigma^2}$ must be violated close to the MBH ($R_{\rm t}$, $R_{\rm Schw}$, $R_{\rm coll}$)
- Steady state with net inward flux of stars and energy (Peebles 1972)
- However well within R_h but far from $R_{\rm t}$, stars should have nearly-isotropic velocities
- Hence, if single-mass: quasi-steady solution takes power-law form (isotropic DF) $f(E) \sim E^p$, $\rho(r) \sim r^{-\gamma}$, with $\gamma = 3/2 + p$
- BW – Detailed kinematic treatment for **single-mass**
- $\gamma = 7/4$ and $p = \gamma - 3/2 = 1/4$ (Bahcall & Wolf 1976)

– Properties of multi-mass systems only very poorly reproduced by single-mass models

– IMFs $\in [0.1, \sim 120] M_\odot$ to first order by two (well-separated) mass scales : $\mathcal{O}(1 M_\odot)$ (MS, WD, Ns) and $\mathcal{O}(10 M_\odot)$ (SBHs)

– Stars with different mass get distributed with different density profile

– Heavies sink, lighter components float outwards (Spitzer 1987, Khalisi, Amaro-Seoane & Spurzem 2007)

– Models extending to 2-mass components; **BW argued heuristically for a scaling relation** $p_L = m_L/m_H \times p_H$ that depends on the star's mass ratio only

No general result on the H's inner slope

No discussion about the dependence of the result on the component's number fractions

– **Two branches for the solution**, parametrized by $\Delta = \frac{D_{HH}^{(1)}+D_{HH}^{(2)}}{D_{LH}^{(1)}+D_{LH}^{(2)}} \approx \frac{N_H m_H^2}{N_L m_L^2} \frac{4}{3+m_H/m_L}$ (measure for H's self-coupling relative to L's) (Hopman & Alexander 2009)

– **The weak branch** – $\Delta > 1$ corresponds to the scaling relations found by BW77

– **The strong branch** – $\Delta < 1$, generalizes the BW77 solution

– Validation of assumptions inherent to the Fokker-Planck (FP) approximation with N-body (Preto & Amaro-Seoane 2010, Amaro-Seoane & Preto, 2010)

(scattering is dominated by uncorrelated, 2-body encounters and dense stellar cusps are robust against ejections)

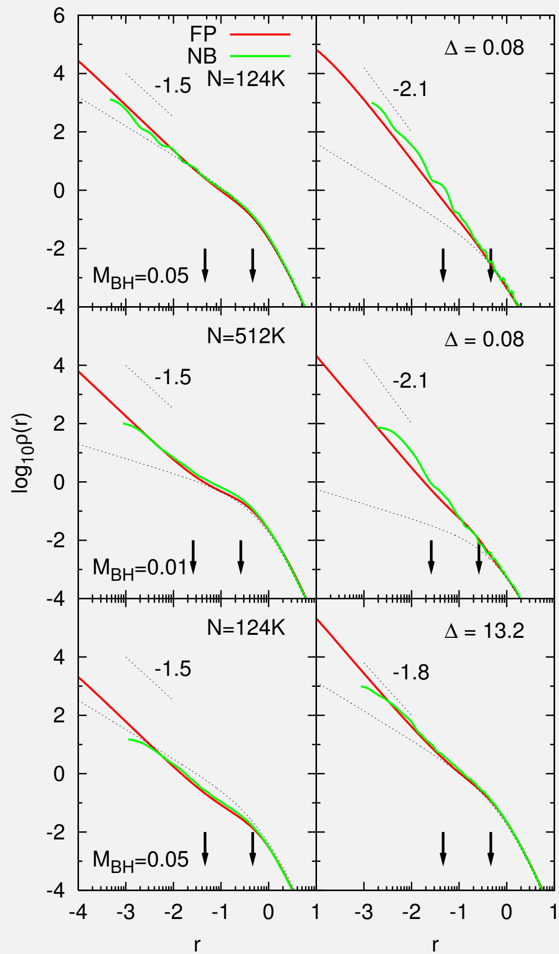
– Not a priori trivial: For a BW $\gamma = 7/4$, stellar velocity high – fraction of stars with speeds close to $v_{\rm esc}$ in cusp very large

– Our FP isotropic, orbit-averaged multi-mass in energy space (similar to Murphy, Cohn & Durisen 1989)

– FP equations is more general than AH09 (only potential MBH) : includes the dynamics of stars unbound to the MBH

– Also Merritt09 : only effect of DF from Ls over Hs, not even H's scattering – **Limited to early evolution** (Hs only minor perturbation on Ls)

- $T=0$ Dehnen profile with $\gamma = 1$ (top & bottom) and $\gamma = 1/2$ (middle)
- $\rho_{\rm L,H}(r)$ (left/right) after $\approx 0.2 T_{\rm rlx}(r_{\rm h})$
- $R = 10$, $f_H = 2.5 \times 10^{-3}$, $f_H = 0.429$
- γ_H from $\gtrsim 2$ to $\approx 7/4$ when moving from strong to weak branch
- $\gamma_L \approx 3/2$ throughout
- Arrows point to r_h and $0.1 r_h$
- **Agreement between both methods is quite good**
- FP validation – Advantages are clear: Much faster calculations



Important implications for regrowth of cusps

Crucial for the estimate of EMRIs

Cusps in distress

- Deficit of old stars based on number counts of spectroscopically identified, old stars in sub-parsec SgrA^* (down to magnitude $K = 15.5$)

  (Do et al. 2009, Buchholz et al 2009)

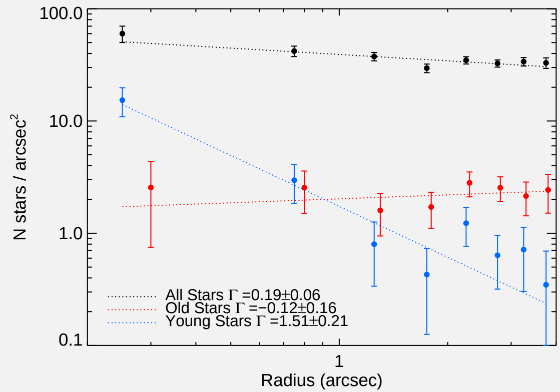
- Best fits seem to favor slopes $\gamma < 1$

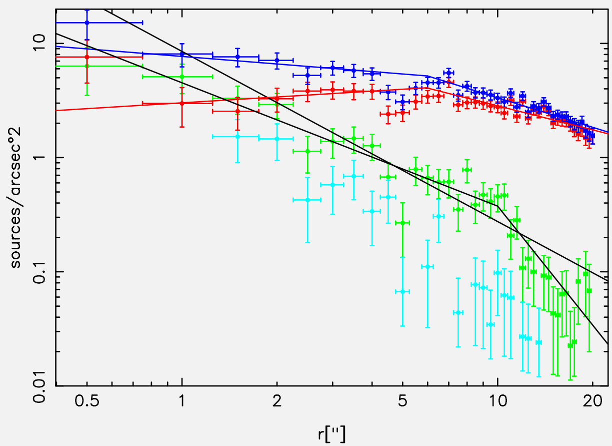
  (Schödel et al 2009)

- Possibility of a core with $\rho_\star$ decreasing, $\gamma < 0$

- Note: detectable stars (essentially late-type giants) are still a small fraction, slope of the density profile is still weakly constrained and such a fit is only marginally better than one with $\gamma \sim 1/2$

- Too early to conclude for the inexistence of a segregated cusp?





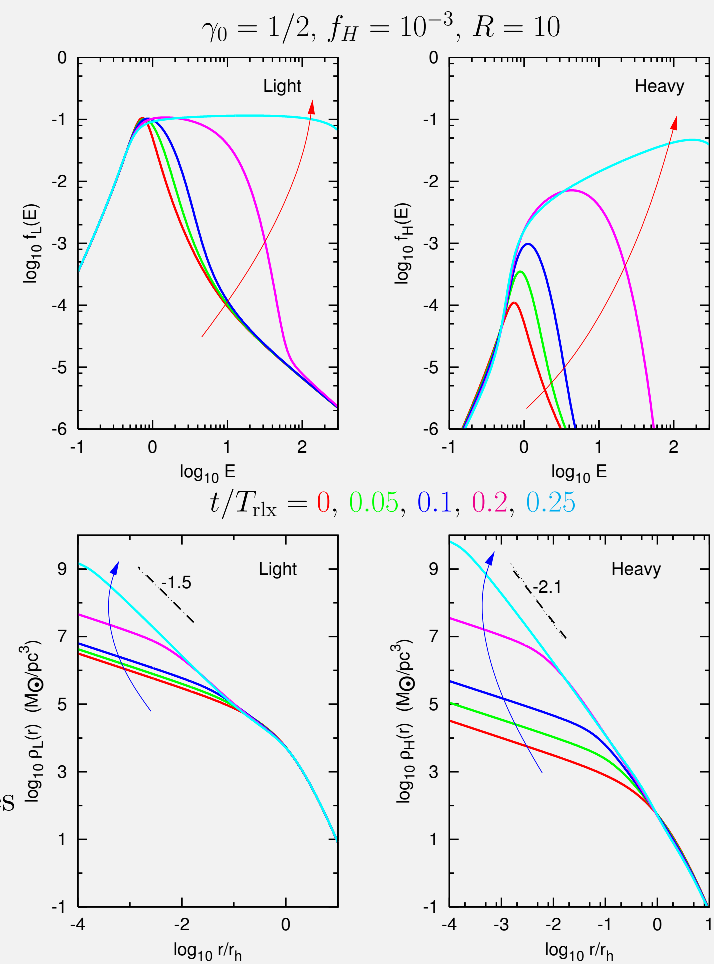
(Do et al 2009, Buchholz et al 2009)

Isocore regrowth

- Time necessary for cusp growth if at some point a central core is carved?
- Choose as initial condition a model with $\gamma = 1/2$ – Isotropization time is $\ll T_{rlx}(r_h)$
  (Merritt 2009)
- Our FP isotropic, orbit-averaged multi-mass in energy space (similar to Murphy, Cohn & Durisen 1989)
- FP equations is more general than AH09 (only potential MBH) : includes the dynamics of stars unbound to the MBH
- Merritt09 : only effect of DF from Ls over Hs, not even H's scattering – Limited to early evolution (Hs only minor perturbation on Ls)

We can follow both weak and strong branches of the solution throughout without restriction

- Evolution of phase-space $f(E)$ and spatial $\rho(r)$ densities
- By $t \sim 0.25\ T_{rlx}(r_h)$, cusps with $\gamma_L \sim 1.5$ and $\gamma_H \sim 2$ ($p_L \sim 0.05$ and $p_H \sim 0.5$) are **fully developed** (~ 0.02 pc if scaled to MW)
- $T_{rlx}^{\rm MW}(2.5\ \rm pc) \sim 24$ Gyr
- If carving event more than 6 Gyr ago, a very steep cusp of SBHs would have had time to re-grow
- Merritt09 overestimates the time by neglecting H-H and H-L scattering (only approximately valid as long as $\rho_H \ll \rho_L$)



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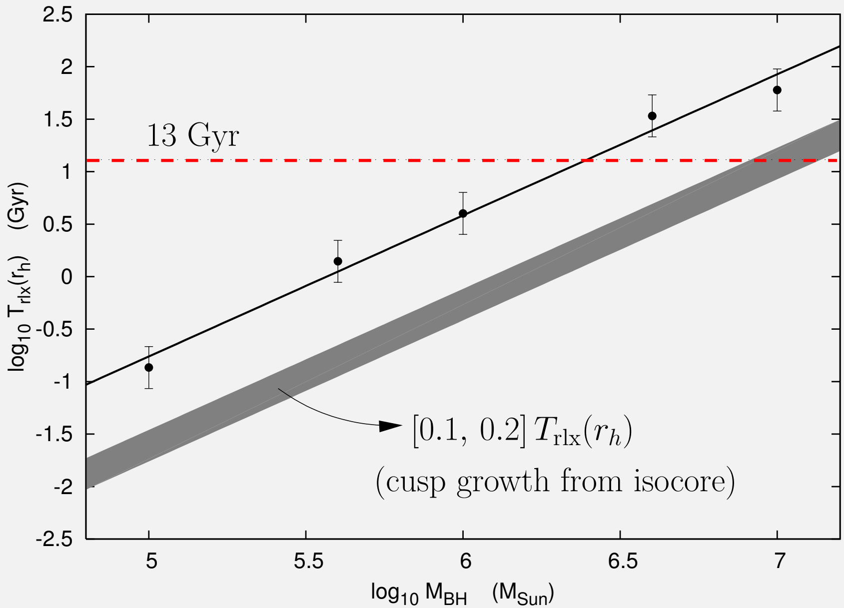


- ▷ $T_{\rm rlx}$ at R_h for LISA masses
- ▷ Straight line: single-mass case
- ▷ Shaded region: $[0.1,\, 0.2]$ – time stellar cusps take to grow starting from isocore
- ▷ *Even* in the case a core was carved, mass segregation enlarges the fraction significantly : more EMRIs sources

Impact for LISA

- (2-b relaxation-driven) **Timescales for cusp growth in a MW type nucleus shorter than** $T_{\rm H}$ ...
- ...unless a very large core/hole is postulated to be present in the initial stellar distribution (*i.e.* a core radius $\gtrsim 2$ pc)
- Best fits from number counts data seem to exclude slopes $\gamma > 1$
- There **could be a core** with a stellar density decreasing towards the center, $\gamma < 0$

Is this a common feature for nuclei in LISA?

- (Rather) bad news for EMRIs
- Which mechanisms could carve out a hole in the cusp?
- IMRIs carve a hole but need a steady inflow of one at roughly every 10^7 years
  (Baumgardt et al 2006, Portegies Zwart et al 2006)
- Another possibility: SgrA^* is a binary MBH – But then there must have been a more or less recent major merger involving the Milky Way

– Stellar cusps may re-grow in less than a $T_{\rm H}$ but the existence of cored nuclei still remains a possibility

– Don't panic. . . LISA EMRI rates peak around $\mathcal{M}_\bullet \sim 4 \times 10^5 - 10^6 M_\odot$ and re-growth times are $\lesssim 1$ Gyr for $\mathcal{M}_\bullet \lesssim 1.2 \times 10^6 M_\odot$

(Amaro-Seoane et al 2011 in prep)

– The Milky Way nucleus is *not* necessarily the prototype of the nucleus from which LISA detections will be more frequent

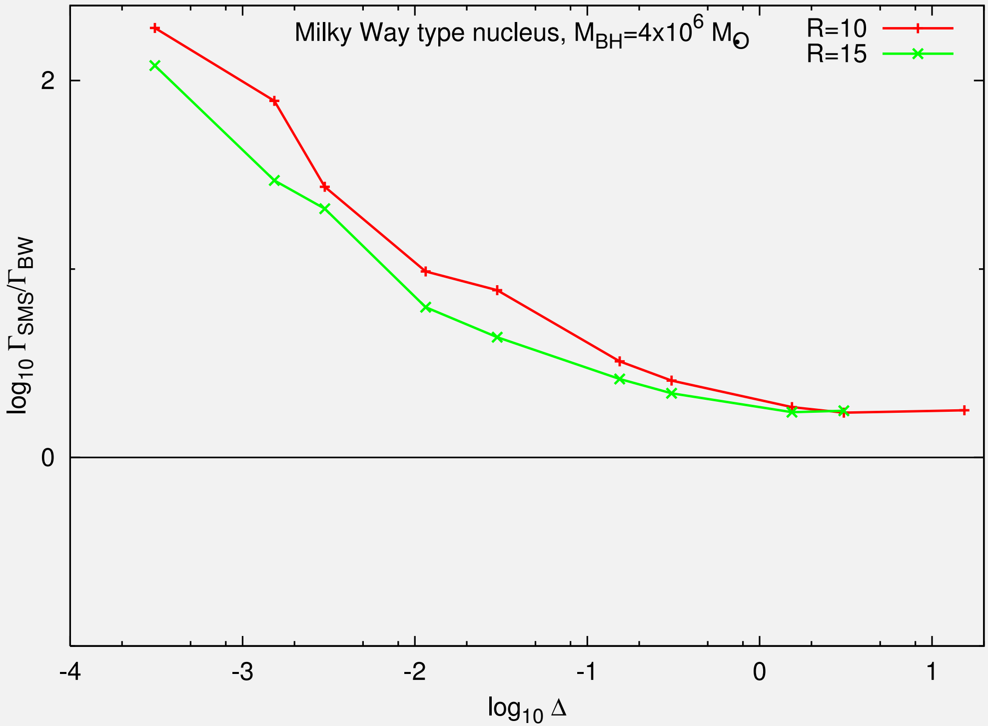
We still expect that a substantial fraction of EMRI events will originate from segregated stellar cusps

– Impact of this on rates?

EMRI event rate estimation

$$\Gamma_{\rm EMRI} = f_\bullet \int_{E_{\rm GW}}^{+\infty} dE \, \frac{n(E)}{\ln(J_c(E)/J_{\rm lc}) \; T_{\rm rlx}(E)}$$

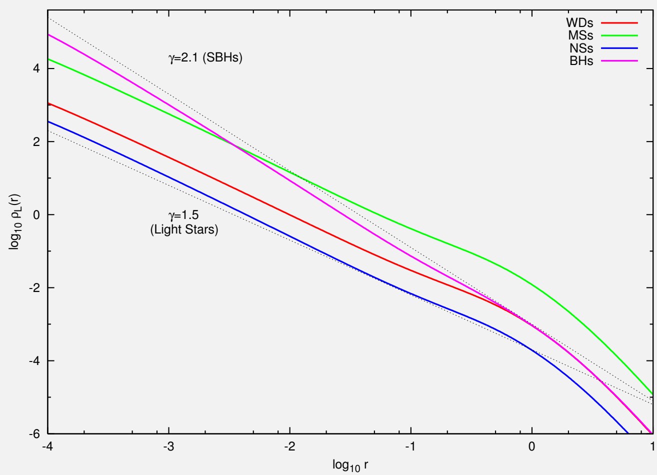
- $f_\bullet$ is the number fraction of SBHs in the stellar population
- $n(E)$ is the number of stars per unit energy
- $J_c(E)$ is the specific angular momentum of a circular orbit of energy E
- J_{lc} is the loss cone angular momentum
- $T_{rlx} = 0.34\ \sigma^3/[G^2(m_\bullet \rho_\bullet + m_* \rho_*) \ln \Lambda]$
- For $r \ll r_h$: $\langle E(r) \rangle = \frac{GM_\bullet}{2r}$ or $E = \frac{GM_\bullet}{2a}$
- a_{GW}, or energy E_{GW}, for EMRIs is: $a_{GW} = 0.01 r_h$
  (Hopman & Alexander 2005, to be checked)
- Rescale with $M_\bullet$ assuming that $M_\bullet \propto \sigma^4$ throughout ($r_h \propto M_\bullet^{1/2}$)

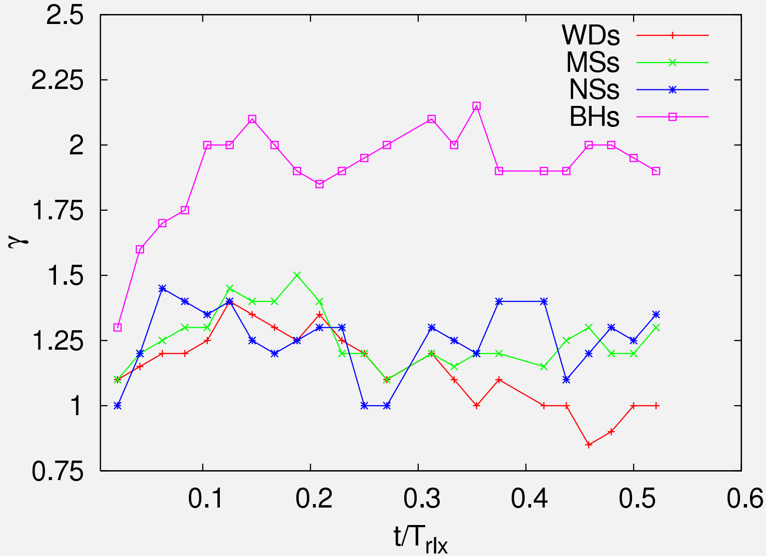


Milky Way type nucleus, M_{BH}=4x10^6 $M_\odot$
R=10
R=15
2
0
-4
-3
-2
-1
0
1
$\log_{10} \Delta$
$\log_{10} \Gamma_{SMS}/\Gamma_{BW}$

Multi-components

- Steady state density profiles 4 comp. after $\approx 0.15 T_{\rm rlx}$
- **WDs** ($f_{\rm WD} = 0.11$ and $R_{\rm WD} = m_{\rm WD}/m_{\rm MS} = 0.6$)
- **MSs** ($f_{\rm MS} = 0.873$ and $R_{\rm MS} = 1$)
- **NSs** ($f_{\rm NS} = 0.01$ and $R_{\rm NS} = m_{\rm NS}/m_{\rm MS} = 1.4$)
- **BHs** ($f_{\rm BH} = 0.007$ and $R_{\rm BH} = m_{\rm NS}/m_{\rm BH} = 10$)
- $\gamma_{\rm BH} \sim 2.1$, while $\gamma_{\rm WD} \sim \gamma_{\rm NS}$: strong branch
- MSs dominate in number everywhere
- But for $r \lesssim R_{eq} \sim (3\text{–}4) \times 10^{-3} \approx 0.005 r_h$, **SBHs dominate in density**





- Temporal evolution of the logarithmic slopes of the spatial densities of all four components
- NB qualitatively similar to the one obtained with FP
- NB segregation is slightly stronger
- SBHs clearly stands out above all the others, a clear sign that SBHs have segregated to the center
- Slopes seem to stabilize around $\gamma_{\rm BH} \sim 2$, $\gamma_{\rm MS} \sim \gamma_{\rm NS} \sim 1.25$

Conclusions

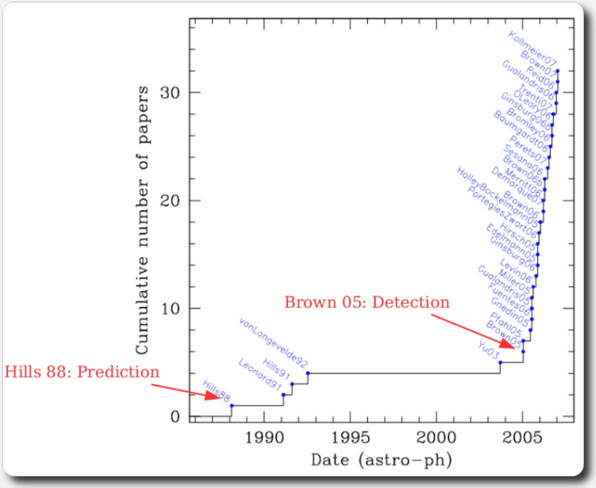
- BW solution for the weak branch is unrealistic : High $\Gamma_{\rm EMRIs}$ because unrealistically high $f_\bullet$ (≥ 0.05)
- In the more realistic case, when $\Delta \sim 0.03$, ($f_\bullet \sim 10^{-3}$) the BW solution would entail a strong supression of the EMRI rate to –at best – a few tens of events per Gyr
- This is where SMS solution appears to rescue us : higher $\rho_\bullet$ well inside r_h
- SMS implies a higher $\rho_\bullet$ well inside r_h
- A boost in diffusion SBHs close to MBH
- SMS compensates for the small $f_\bullet$ that come from *unrealistic* IMFs

When going from $f_\bullet^{\rm BW}$ high (unrealistic, say $\Delta = 3$) to realistic ($\Delta = 0.03$) $\Gamma_{\rm EMRIs}$ supressed by factors of $\sim 100 - 150$ if SMS ignored

- From $\Gamma \sim$ few $\times 10^3$/Gyr to $\sim$ few tens/Gyr
- By taking into account SMS, for this low $\Delta = 0.03$, we boost the rates from few tens to a few hundred per Gyr, ~ 250/Gyr if $R = 10$ and $f_\bullet = 0.001$

Tidal separation

Binary enc. with SMBH: Zero-eccentricity LISA events



A stampade!

- Studies of EMRIs focus on close pass (CO plunging to the central SMBH + GR + shrinkage of orbit)
- Capture orbits → pericentric distances *very close* to SMBH: Despite final circularization, $e|_{\rm LISA} \sim 0.5 - 0.9$
- Stellar-mass binary with a CO which will be *tidally separated* ⇝ one bound object + one ejection

- No energy needs to be dissipated to have a capture $\rightarrow$
- Immune to perturbations
- Binary with total mass $m + a$ plunging towards SMBH of mass $\mathcal{M}_{\bullet}$ separated if comes closer than
  $7\ \mathrm{AU}(\mathcal{M}_{\bullet}/10^6\,M_{\odot})^{1/3}(m/10\,M_{\odot})^{-1/3}(a/0.1\ \mathrm{AU})$
- Depending on a, $r_{\rm per} \rightsquigarrow \Sigma$ could be $10^2 - 10^3$ larger
- $\mathrm{Rate}|_{\rm binary} = 1 - 2$ orders of magn.$\times$ $\mathrm{Rate}|_{\rm single}$
- If more than $\sim 1 - 10\%$ of COs are in binary ...

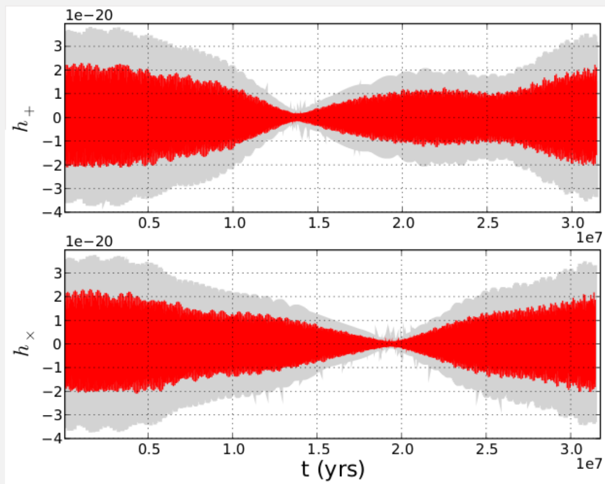
EMRIs dominated by tidal separation events

- What's more:

$\sim$80-90% of the potential –traditional– EMRI events are lost

- COs candidates for LISA (not direct plunges) have large apoapsis: Perturbations will scatter them out

- This is not the case of BCOs (apocenter distance is usually only tens of times the pericenter distance)
- $t_{\text{modif. of per.}} \ll t_{\text{orb}}$
- Perturbations will be gradual $\rightarrow$ Circularization
- Waveforms typical EMRI vs binary EMRI (full yr of data)
- Final plunge at $r_{\text{plunge}} \equiv 2 \times r_{\text{ISCO}}$
- $M_{\bullet} = 3 \times 10^6\, M_{\odot}$
- Spin $a = 0.5\, M$
- Neglect the spin of the star
$$\mathcal{M} := 1 - \frac{\langle h_{\text{EMRI}} | h_{\text{bin}} \rangle}{\sqrt{\langle h_{\text{EMRI}} | h_{\text{EMRI}} \rangle \langle h_{\text{bin}} | h_{\text{bin}} \rangle}}$$
- Missmatch of 99.9971%



Plunges

–a.k.a "disguessed EMRIs"–

. . . (Work in progress with Carlos F. Sopuerta and Miguel Preto)

- Previous works about the estimation of how EMRI events differ in orders of magnitude
- A common result to all investigations is that the probability that a compact object merges with the MBH after only one intense burst of GWs is much more likely than a slow inspiral, an EMRI
- The later is refered to as a "plunge" because the compact object dives into the MBH, crosses the horizon and is lost as a probe of strong gravity for LISA
- The event rates for plunges are overwhelming as compared to slow inspirals.
- Nevertheles . . . "pure plunges" simply ***do not exist***
- Nature MBH's are spinning and the magnitude of the spin has been sized up to be high (though see recent work by A. King 2009)
- Number of periapsis passages that a compact object set on to an extremely radial orbit goes through before being actually swallowed by the Kerr MBH?

- By extracting the information carried by the GWs, we can determine the mass and spin of the central MBH with unprecedented precision and we can determine how the holes "eat" compact objects that happen to be near them

- Even if it was a "flashing source" and not a real EMRI, not as descriptive and appealing as an EMRI, the information contained in the waveform of a repeatedly bursting system is huge

- Next slide: results of calculation

- Take initial orbital parameters (p, e, i), calculate constants of motion (E, L_z, C), then the average flux of these "constants", i.e. the average time evolution ($\dot{E}$, $\dot{L}_z$, $\dot{C}$)

- Calculate time to go from apo to periapsis and back (radial periode) and thus the change in (E, L_z, C) and so the new constants of motion, therefore: ($p_{\rm new}$, $e_{\rm new}$, $i_{\rm new}$) until we find the parameters corresponding to a plunge or unstable orbit

$\mathcal{M}_\bullet$	Spin (a/M)	a_0 (pc)	e_i	i (rad)	$\tau_{\rm mrg}$ (yrs)	$\tau_{\rm LISA}$	Peri (LISA)
3E6	0.990	8.6182E-4	0.9990	0.6	2.6755E3	6.8409E2	432503
1E6	0.990	2.8727E-4	0.9990	0.6	2.9743E2	1.1915E2	146074
1E6	0.500	2.8727E-4	0.9990	0.6	2.4714E2	9.8328E1	97715
3E6	0.500	8.6182E-4	0.9990	0.6	2.2229E3	5.6105E2	288372
1E6	0.900	2.3939E-4	0.9990	0.2	1.5328E2	6.8038E1	90555
3E6	0.900	7.1818E-4	0.9990	0.2	1.3785E3	3.9237E2	268423
3E6	0.900	7.1786E-3	0.9999	0.2	4.6101E3	3.9131E2	267802
3E6	0.900	5.7429E-3	0.9999	0.2	2.0757E3	1.9956E2	149747
3E6	0.900	5.0250E-3	0.9999	0.2	1.3164E3	1.3607E2	106563
1E6	0.900	1.6750E-3	0.9999	0.2	1.4843E2	2.3449E1	35889
1E6	0.900	1.4357E-3	0.9999	0.2	9.1260E1	1.5533E1	24593
1E6	0.900	1.4357E-3	0.9999	0.1	9.2711E1	1.5769E1	25038
3E6	0.900	4.3071E-3	0.9999	0.1	8.1857E2	9.1641E1	74371
5E6	0.900	7.1786E-3	0.9999	0.1	2.2652E3	2.0548E2	122993
1E6	0.900	1.4357E-3	0.9999	0.1	1.8272E2	3.1556E1	50075
4E6	0.700	6.7000E-3	0.9999	0	1.8937E3	1.7207E2	96284
4E6	0.998	6.7000E-3	0.9999	0	2.6993E3	2.4753E2	170494
4E6	0.998	9.5714E-3	0.9999	0	8.7952E3	6.6162E2	395248
4E6	0.998	7.6571E-3	0.9999	0	4.1097E3	3.5062E2	230973
4E6	0.998	6.7000E-3	0.9999	0	2.6993E3	2.4753E2	170494
4E6	0.998	5.7429E-3	0.9999	0	1.7598E3	1.7468E2	123868
4E6	0.998	5.7429E-3	0.9999	0.3	1.6574E3	1.6506E2	117974

Note: Prograde orbits, retrograde on-going; $m_\bullet = 10\,M_\odot$

Many periapsis passages but over a long time (many years). When LISA is "switched on" only a subset will be seen. Two possibilities:

- ⋆ **Not enough cycles for detection and parameter extraction, potential risk of having a strong GW background**
- ⋆ **Significant contribution to LISA EMRIs: Requires postprocessing (not today!): Calculate SNR etc (needs waveforms)**

Analytical rates

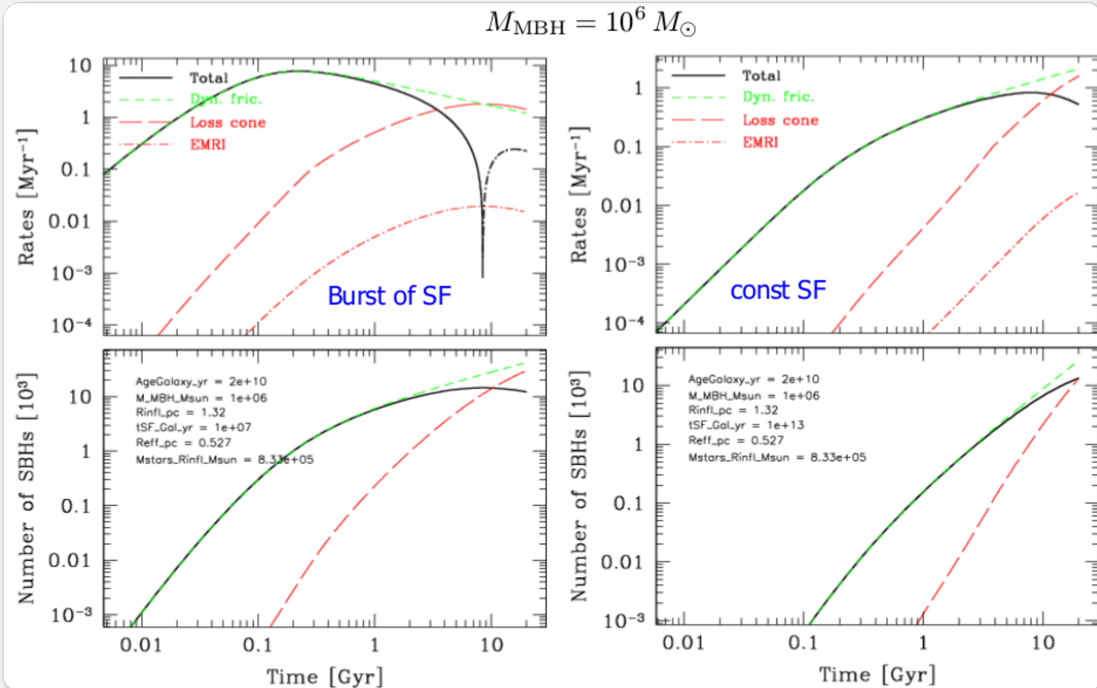
- Stellar-mass black holes (SBHs) form throughout galactic central region – Assume no stellar formation in the vicinity of MBH (influence radius)
- SBHs drift to the centre through mass segregation – Mass segregation treated as pure dynamical friction on circular orbits
- SBHs form power-law density cusp around MBH $n(R) \propto R^{-\gamma}$ (but see talk by David and Rainer!)
  –Radius of cusp about $0.2 - 1 \times R_{\rm infl}$
  –Lighter, "background" stars form shallower cusp
  –If $\gamma = 1.5$, relaxation time is indep of R
- In cusp, orbits are modified by various processes
  –2-body relaxation
  –emission of gravitational waves
  –resonant relaxation (neglected)
- Below $a_{\rm GW}$, $\tau_{\rm GW} < T_{\rm rlx} < \tau_H$ and objects inspirals as EMRIs
  –Above $a_{\rm GW}$, objects survive or are swallowed on "direct-plunge orbits" (not strictly true)

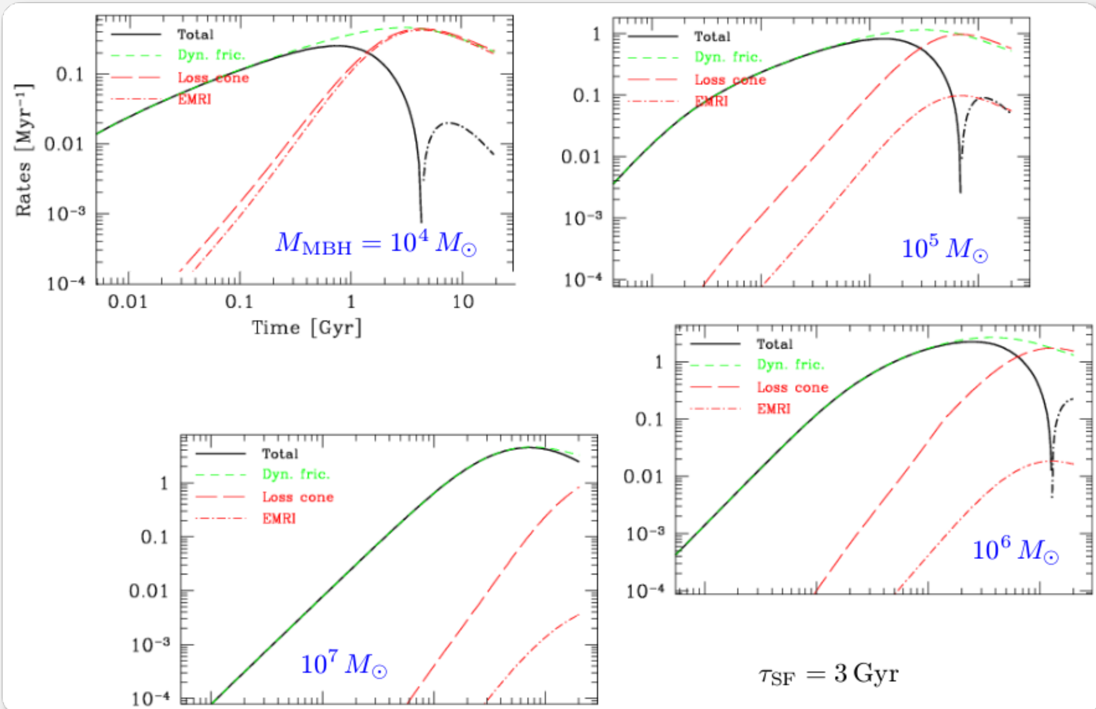
Outline
EMRIs
Distribution of stars
Cusps in distress
Impact for LISA
Multi-components
Conclusions
Tidal separation
Plunges
Analytical rates
Close

- Formation of stellar BHs: Assume formation rate $\propto e^{-t/\tau_{\rm SF}}$
- Inspiral by dynamical friction
- Swallowing by MBHs: EMRIs below $a_{\rm EMRI}$, direct plunges above $a_{\rm EMRI}$ $\dot{N}_{\rm LC} = \dot{N}_{\rm EMRI} + \dot{N}_{\rm plunge}$
- Optional: "in-situ formation" of stellar BHs $\dot{N}_{\rm insitu}$
- Stellar formation in an accretion disc?
- Deposition of massive stars and BHs by tidal disruption of binaries?

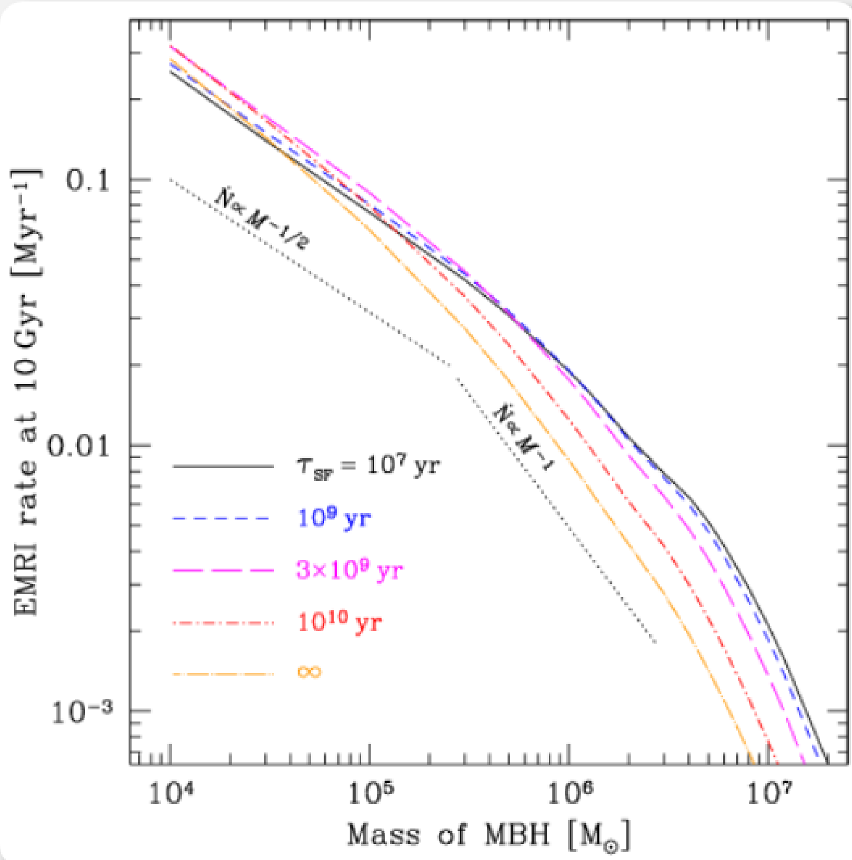
$$\dot{N}_{\rm SBHS,\,cusp} = \dot{N}_{\rm DF} - \dot{N}_{\rm LC} + \dot{N}_{\rm in\ situ}$$

Note: $\dot{N}_{\rm LC} \propto t_{\rm rlx}^{-1}\, N_{\rm SBHs} \propto (m_{\rm SBHs}^2 N_{\rm SBHs} + m_{\rm MS}^2 N_{\rm MS}) N_{\rm SBHs}$ Warning: $\gamma = 1.5$ assumed for all objects in cusp


$M_{\rm MBH} = 10^6\,M_\odot$
Total
Dyn. fric.
Loss cone
EMRI
Burst of SF
const SF
Rates [Myr⁻¹]
Number of SBHs [10³]
AgeGalaxy_yr = 2e+10
M_MBH_Msun = 1e+06
Rinfl_pc = 1.32
tSF_Gal_yr = 1e+07
Reff_pc = 0.527
Mstars_Rinfl_Msun = 8.33e+05
AgeGalaxy_yr = 2e+10
M_MBH_Msun = 1e+06
Rinfl_pc = 1.32
tSF_Gal_yr = 1e+13
Reff_pc = 0.527
Mstars_Rinfl_Msun = 8.33e+05
Time [Gyr]
Time [Gyr]



Total
Dyn. fric.
Loss cone
EMRI
Rates [Myr⁻¹]
Time [Gyr]
$M_{\rm MBH} = 10^4\,M_\odot$
$10^5\,M_\odot$
$10^6\,M_\odot$
$10^7\,M_\odot$
$\tau_{\rm SF} = 3\,{\rm Gyr}$



0.1
0.01
10^{-3}
EMRI rate at 10 Gyr [Myr^{-1}]
10^4
10^5
10^6
10^7
Mass of MBH [$M_\odot$]
$\dot{N} \propto M^{-1/2}$
$\dot{N} \propto M^{-1}$
$\tau_{SF} = 10^7$ yr
10^9 yr
3×10^9 yr
10^{10} yr
∞

Relaxed nuclei

- LISA will observe EMRIs for $10^4 M_\odot \lesssim \mathcal{M}_\bullet \lesssim 10^7 M_\odot$
- Assume $\mathcal{M}_\bullet - \sigma$, $\mathcal{M}_\bullet \propto \sigma^4$
- For an isothermal cluster ($\rho \sim r^{-2}$ and $\sigma(r) = \text{const}$, for $r \gtrsim \text{few} \times 0.1 \mathrm{r_h}$),

$$r_h \sim \frac{G\mathcal{M}_\bullet}{\sigma^2} \propto \mathcal{M}_\bullet^{1/2},$$
$$\langle n \rangle|_h \sim \frac{2\mathcal{M}_\bullet/m}{r_h^3}$$

- Jeans equation : $\sigma^2(r) \sim G\mathcal{M}_\bullet/r$ (for $r < r_h$),

$$T_{\rm rlx} \sim \frac{\sigma^3}{\rho} \propto \mathcal{M}_\bullet^{5/4}.$$

- **Rule of thumb:** Nuclei harboring MBH with $\mathcal{M}_\bullet \gtrsim 10^8 M_\odot$ have $T_{\rm rlx} > T_H$, where $T_H \sim 13 - 14$ Gyr
- Their stellar distributions retain memory from the formation process and/or the latest strong perturbation
- Those with $\mathcal{M}_\bullet \lesssim 10^6 M_\odot$ have $T_{\rm rlx} < T_H$ and should have had time to relaxed into a steady-state
- The Milky Way nucleus, with $\mathcal{M}_\bullet \sim 4 \times 10^6 M_\odot$, stands on the borderline

Validation

- There exist N-body realisations of single-mass solution (Preto et al 2004, Baumgardt et al 2004)
- Validation of assumptions inherent to the Fokker-Planck (FP) approximation
  (scattering is dominated by uncorrelated, 2-body encounters and dense stellar cusps are robust against ejections)
- Not a priori trivial: For a BW $\gamma = 7/4$, stellar velocity high – fraction of stars with speeds close to $v_{\rm esc}$ in cusp very large
- Assuming DF $f(E) \propto (E/\sigma^2)^p$ of a BW cusp, this fraction $N(> v_{esc})/N_{tot}$ with velocity above any given fraction $\alpha \in [0,1]$ of v_{esc}

$$\frac{N(>\alpha\ v_{\rm esc})}{N_{\rm tot}} = 1 - \frac{\int_0^{\alpha v_{\rm esc}} dv\ v^2 \left(\frac{GM_\bullet}{r} - \frac{1}{2}v^2\right)^p / \sigma^{2p}(r)}{\int_0^{v_{esc}} dv\ v^2 \left(\frac{GM_\bullet}{r} - \frac{1}{2}v^2\right)^p / \sigma^{2p}(r)} =$$
$$1 - \frac{\alpha^3\ {}_2F_1(3/2, -p, 5/2; \alpha^2)}{{}_2F_1(3/2, -p, 5/2; 1)},$$

- If $p = 1/4$, $\approx 84\%$ stars with velocity $> v_{esc}/2$ and $\approx 19\% > 0.9 v_{esc}$; if $p = 0$ ($\gamma = 3/2$), then $\approx 87\%$ and $\approx 22.6\%$
- $N-$body show unambiguously that the stellar cusps are robust against stellar ejections

- Are **multi-mass** stellar cusps (weak/strong), obtained from the solution of FP robust against ejection of stars from the cusp?

- The presence of a mass spectrum leads to an increased rate of stellar ejections from the core of a globular cluster (w/o BH)
(Hénon 1969)

- Ejections—due to strong encounters—are *a priori* excluded from FP

- In the limit where the number fraction of heavy stars is realistically small, a solution obtains with density scaling as

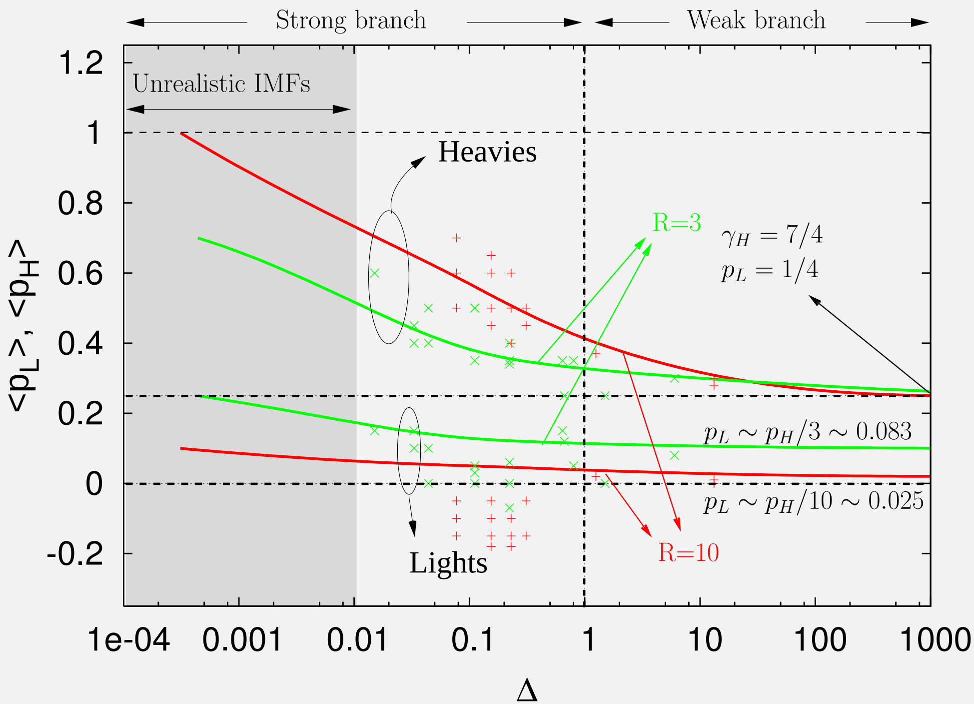
$$\rho_H(r) \sim r^{-\alpha}, \textbf{ where } \alpha \gtrsim 2$$

(Alexander & Hopman 2009)

- Surprisingly small number of multi-mass N-body studies around a MBH
(Baumgardt et al. 2004, Freitag, Amaro-Seoane & Kalogera 2006)

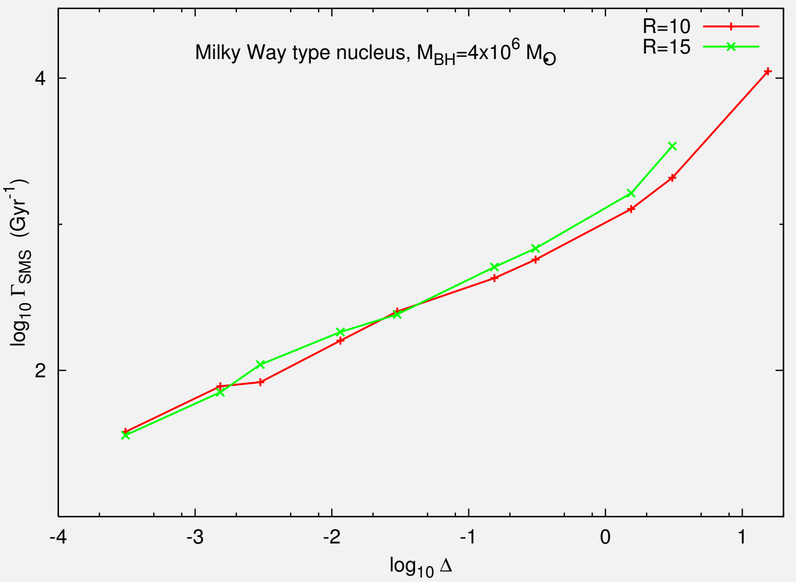
- Are multi-mass FP cusps robust against ejections?
(Hénon, 2969)

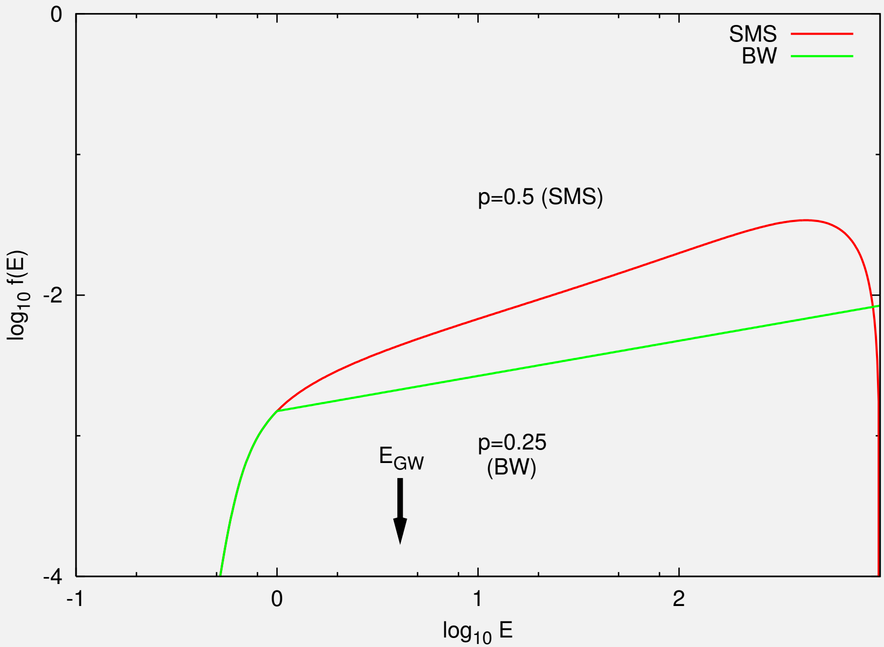
- Fundamental to address with $N-$body



- ⋆ Strong mass segregation is a robust outcome of the growth of stellar cusps around a MBH when $\Delta < 1$
- ⋆ BW77 solution is recovered when $\Delta > 1$
- ⋆ Rate of stellar ejections too low to destroy the high density cusps around MBHs

SMS in energy space and role of Δ





SMS
BW
p=0.5 (SMS)
p=0.25
(BW)
E_{GW}
0
-2
-4
-1
0
1
2
$\log_{10} f(E)$
$\log_{10} E$

EMRI event rate estimation

$$\Gamma_{\rm EMRI} = f_\bullet \int_{E_{\rm GW}}^{+\infty} dE \, \frac{n(E)}{\ln(J_c(E)/J_{\rm lc}) \; T_{\rm rlx}(E)}$$

- $f_\bullet$ is the number fraction of SBHs in the stellar population
- $n(E)$ is the number of stars per unit energy
- $J_c(E)$ is the specific angular momentum of a circular orbit of energy E
- J_{lc} is the loss cone angular momentum
- $T_{rlx} = 0.34\ \sigma^3/[G^2(m_\bullet\rho_\bullet + m_*\rho_*)\ln\Lambda]$

The conversion between r and E is, for $r \ll r_h$: $\langle E(r)\rangle = \frac{GM_\bullet}{2r}$ or $E = \frac{GM_\bullet}{2a}$. The critical radius a_{GW}, or energy E_{GW}, for EMRIs is: $a_{GW} = 0.01 r_h$ from Hopman & Alexander (2005) and is, to first order, independent of $M_\bullet$ (Hopman 2009). This is to be checked independently, and refined if need be, in the papers that follow. The log term in the equation arises from the phase space depletion due to the presence of a loss cone, and one uses $J_c(E) = \sqrt{GM_\bullet}/2E$ and $J_{lc} = 4GM_\bullet/c$. The model quantities are rescaled with $M_\bullet$ assuming that $M_\bullet \propto \sigma^4$ throughout, which means that $r_h \propto M_\bullet^{1/2}$. The mass of the cluster is assumed to be $M_{cl} \propto M_\bullet$. In order to evaluate the boost Γ_{SMS}/Γ_{BW} to the EMRI rates from SMS, for a given Δ, one needs to estimate what would be the rate ifthe spatial and phase space densities were given by the 7/4 Bahcall & Wolf cusp for $r \ll r_h$, while keeping the same densities at r_h. This is done as follows:

$$\rho(r) = \rho_{FP}(r), \qquad r > r_L$$
$$\rho(r) = \rho_{FP}(r_L) \times \left(\frac{r_L}{r}\right)^{7/4}, \qquad r \le r_L,$$

and

$$f(E) = f_{FP}(E), \qquad E < E_L$$
$$f(E) = f_{FP}(E_L) \times \left(\frac{E}{E_L}\right)^{1/4}, \qquad E \ge E_L,$$

where the indices FP mean that the profile is taken from the FP calculation.