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The moment of inertia of main sequence stars

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Summary. — We present a grid of values for the moment of inertia and the potential energy of hydrogen-burning stars as derived from evolutionary models computed with updated input physics and opacities. A range from 1.0 to 25 solar masses is covered using a standard chemical composition $(X, Z) = (0.70, 0.02)$ together with a value of 2.0 for the mixing-length ratio. Values of the gyration radius and the potential energy of each individual configuration are tabulated in a format suitable for the use in dynamical studies of binary stars. A comparison with previous computations, which adopted very simplified structures, as well as variations with respect to mass, age and chemical composition, are discussed in some detail.

Key words: evolutionary models — internal structure — moment of inertia — apsidal motion.

1. Introduction.

One of the most exciting topics in the study of close binary systems is represented by tidal effects. A star that belongs to such a system is perturbed by the presence of the other component and its basic properties may be changed drastically if they are close enough, through mass transfer, or may at least show a special dynamical behaviour depending on its internal structure. For example, those distortions in the shape of the stellar surface which appear as a consequence of the proximity between the two components, cause a slow rotation of the line of the apses in eccentric binaries. Such an effect is known as apsidal motion and provides one of the few ways to have an empirical insight into the interior of the stars. Observationally, apsidal motion periods can be accurately measured in some eclipsing binaries by monitoring the relative displacement of the primary and secondary eclipses with respect to each other.

Another well-known method to probe stellar interiors is to investigate the behaviour of close binary stars in terms of their actual eccentricities and rotational velocities, i.e. the level of circularization and synchronization achieved in each individual case for a particular age and structure. If we observe a system to have a circular orbit, we are able to compare its evolutionary age, as derived from absolute dimensions, with the circularization time scale given by tidal evolution theories. Of course, ages derived from theoretical evolutionary models behave as lower limits to tidal theories, that is, if the system shows a circular orbit, they should be at least equal to theoretical circularization time. Several throughful investigations have been made about tidal evolution in close binaries (Zahn, 1977 ; Kopal, 1978 ; Hut, 1981 ; Tassoul, 1988) and a general feature is that circularization time scales are obviously dependent on the moment of inertia of the involved stars.

On the other hand, synchronization of the rotational period of the star with orbital values is another predictable consequence of tidal evolution theories. Nevertheless, it is much more difficult to compare theoretical and observational results. While it is straightforward to determine whether a binary system has an eccentric orbit, rotational velocities are rather difficult to measure with the required accuracy and, furthermore, we can never be sure that the observed surface values do actually correspond with that of the stellar interior. Current theories as well as stellar seismology techniques show, despite the mentioned difficulties, that differential rotation in the interior should not be large and a general agreement between observed rotational velocities at the surface of the components of close binaries with orbital values is known to exist (e.g., see Plavec, 1968). Of course, tidal evolution theories again require a good knowledge of stellar moment of inertia to predict synchronization time scales.

We should also remember here that the average level of synchronization along the stellar interior is an important parameter in the computation of the rotational term to the apsidal motion rate of eccentric binaries. In addition, for

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non circular orbits, the concept of circularization should be modified since the orbital velocity is not constant. Different tidal theories predict different values of pseudo-synchronization which, on the other hand, should be in principle compatible with actual observations.

Tabulated values of the moment of inertia for reliable stellar configurations are, unfortunately, difficult to find. Motz (1952) published the computation of a grid of moments of inertia for models available at that time. These were of course extremely simple and even unrealistic. He adopted the internal structure models by Epstein (1951) and Hen and Schwarzschild (1949), and also polytropes for different values of the index n. In order to discuss the rotational properties of low-mass stars, Ruciński (1988) computed fractional radii of gyration using the composite-polytrope model of Rappaport *et al.* (1983). Using the size of the radiative core as derived from models computed by VandenBerg, he could obtain realistic values for the radius of gyration of zero-age main-sequence stars with masses from 0.4 to 1.2 $M_{\odot}$. Calculations for more massive stars, or a direct integration of the internal structure of main-sequence stars, has not been yet published. A modern grid of models is clearly needed and we present the results of such a study in this paper.

A current procedure to calculate circularization and synchronization time scales is to adopt the radius of gyration as a constant (Press *et al.*, 1975 ; Tassoul, 1988) with value 0.1, for a chemically inhomogeneous model, or 0.2, for the case of a polytrope of index $n = 1.5$. These values are taken from Motz (1952) but, of course, the moment of inertia changes while the star evolves. Therefore, adopting a single value of the gyration radius, moreover based upon not physically reliable models, is certainly not the best procedure. In the present paper, we present new computations of the radius of gyraton, as well as the potential energy, for a variety of stellar configurations based upon an updated grid of evolutionary models during hydrogen burning phases.

2. Physics of the models.

The stellar models were based upon the standard assumptions that stars can be described by spherically symmetric configurations, that there is no mass loss, that zero-age main sequence (ZAMS) stars are homogeneous, that regions in radiative equilibrium are always stable, while convective zones are fully mixed. The calculations were made with a modified and updated version of the computer code by Kippenhahn *et al.* (1967). The latest opacity library from Los Alamos (Huebner *et al.*, 1977) and reaction rates as revised by Harris *et al.* (1983) were adopted. A standard chemical composition of $(X, Z) = (0.70, 0{,}02)$ was taken as well as a mixing-length parameter of 2.0. Additional chemical compositions were used as indicated by Claret and Giménez (1989) to check for the effect of changes in helium and/or metal content. The grid covers a range from 1 to 25 solar masses.

A more detailed description of the physics of the interior models used to compute the different stellar parameters presented in this paper, is given by Claret and Giménez (1989).

3. The grid.

The moments of inertia were computed according to the definition,

$$I = \frac{8\pi R^2}{3} \int_0^R \rho(r)\, r^4 \mathrm{d}r$$

while gravitational energy is given by

$$\Omega = -G \int_0^M \frac{m(r)}{r} \mathrm{d}m(r) = -\alpha\, G\, \frac{M^2}{R}$$

and the radius of gyration is, of course, defined as

$$\beta = (I/MR^2)^{1/2}$$

where in all equations above, M and R denote the mass and radius of the adopted configuration and ρ is the local density.

A Runge-Kutta method of fourth order was adopted for all numerical integrations. Due to the large number of layers given by theoretical models, changes in the integration step did not affect the results. All computations were carefully checked with known mass distributions such as polytropes, rigid spheres, etc.

In table I, we show the obtained values of the previously mentioned parameters for stellar models in the zero-age main sequence (ZAMS). Evolutionary tracks are given in tables II.1 to II.17 and include also the age in logarithm.

4. Comparison with previous models.

For the ZAMS models in table I, a linear relation between the radius of gyration and the apsidal motion constant, k_2, is found as shown in figure 1 and expected from earlier works by Motz (1952) and Ureche (1976). However, it can be seen in figure 2 that the behaviour of β as a function of mass is clearly different for masses larger or smaller than 1.5 solar units, corresponding to the change in the dominant energy source from the CNO cycle to the proton-proton chain. The same effect is also noticed in the $\log k_2 - \log M$ plane as shown by Hejlesen (1987) and they seem to partly compensate each other allowing a straight estimation of the radius of gyration from the internal structure constant. In order to allow a direct comparison of results, computations by Motz are also included in figure 1. These values correspond nevertheless to very different configurations, those in the upper right corner being approximately valid for inhomogeneous giants. Our values correspond, all of them to homogeneous ZAMS stars. Furthermore, the linear

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relation shown in figure 1 is kept even if we include evolved models during core hydrogen burning phases though the scatter around the mean relation becomes slightly larger.

The linear coefficients corresponding to each of the relations plotted in figure 1, are rather similar though, as commented below, the behaviour of the radius of gyration when the star evolves outside the main sequence is quite more complex. This result shows the overall character of the relation between $\log k_2$ and $\log \beta$.

It should be pointed out that, despite previous papers by Motz (1952) and Ureche (1976), we give moments of inertia as a function of directly observable parameters in double-lined eclipsing binaries in table II. In figure 3 we have plotted the tabulated values in the $\log g - \log \beta$ plane for individual evolutionary tracks computed for some selected masses.

When comparing our results with the calculations made by Ruciński (1988) on the base of the composite-polytrope model, we found that direct integration provides smaller values of the parameter β. Taking our model of 1 solar mass as representative of the low-mass stars studied by Ruciński, the fractional mass contained within the radiative core is found to be 0.982 for the ZAMS. This represents, for the composite-polytrope model of indices 3 (inner) and 1.5 (outer), a value of $\beta = 0.325$ while direct integration of the definition integral given above results in a value 0.277 (see Tab. I). Estimations made by Ruciński on the base of models by VandenBerg, for the depth of the convective zone, are in excellent agreement with our results. It seems therefore that the composite-polytrope formalism is not accurate enough for our purposes.

5. The gravitational energy.

With respect to the potential energy, we should comment that it also shows a good correlation with the internal structure coefficient. In fact, during the main sequence, this relation is clearly linear in such a way that the product $\alpha\beta$ remains almost constant. However, when the stars evolve outside the main sequence a different approach is required and we have found that a third order polynomium describes satisfactorily the relation between $\log k_2$ and $\log \alpha$.

The computation of the parameter α, giving the gravitational energy of each configuration, is very important in the study of Jacobi's equation and the interpretation of a variety of symmetries in stellar structure (Ferronsky *et al.*, 1987). It is convenient to point out that, although all the computations made for this paper are only valid in spherical configurations, in the case of elliptical symmetry and ellipsoidal mass distribution, α and β can be written in terms of the values for the spherical structure and correction factors due to the geometry of the perturbation. Some interesting applications of these structural properties are given by Ferronsky *et al.* (1987, Chapters 6 and 7).

6. Conclusions.

The present paper is of special relevance for studies of binary stars. Internal structure coefficients of stars in the zero-age main-sequence show a tight correlation with the corresponding radii of gyration while, for the first time, we show the important effect of evolution on the moment of inertia for given masses. Results for other chemical compositions, around the standard mixture adopted, show identical behaviour and the individual values of the gyration radius and potential energy are not significantly different from those given in table II. We found therefore unnecessary to include further tabulations. A grid of values for the potential energy during hydrogen burning phases is also given. Simplified methods to estimate these important parameters in the understanding of the dynamical behaviour of main-sequence stars are superseded by the present grid through direct integration of stellar models.

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TABLE I.

Log m	Log g	Log Teff	Alpha	Beta
0.00	4.4900	3.7562	1.531	0.277
0.05	4.4178	3.7838	1.712	0.248
0.10	4.3540	3.8110	1.890	0.224
0.15	4.3124	3.8450	2.020	0.209
0.20	4.3220	3.8865	2.006	0.209
0.30	4.3458	3.9668	1.907	0.218
0.40	4.3505	4.0396	1.816	0.227
0.50	4.3460	4.1085	1.736	0.236
0.60	4.3366	4.1750	1.664	0.245
0.70	4.3276	4.2393	1.598	0.253
0.80	4.3147	4.3003	1.543	0.261
0.90	4.3012	4.3584	1.493	0.269
1.00	4.2880	4.4125	1.449	0.276
1.10	4.2759	4.4623	1.411	0.283
1.20	4.2652	4.5082	1.379	0.289
1.30	4.2557	4.5498	1.352	0.294
1.40	4.2474	4.5870	1.330	0.299

TABLE II.1.

Log m = 0.0000

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.4900	3.7562	1.531	0.277
8.8451	4.4765	3.7577	1.560	0.273
9.4001	4.4368	3.7612	1.648	0.263
9.5756	4.4036	3.7633	1.721	0.255
9.6612	4.3787	3.7643	1.776	0.250
9.7830	4.3246	3.7650	1.892	0.241
9.9069	4.2242	3.7628	2.113	0.229
9.9513	4.1513	3.7592	2.283	0.224
9.9675	4.1184	3.7564	2.349	0.224
9.9850	4.0642	3.7507	2.462	0.225
9.9958	4.0251	3.7450	2.519	0.230
10.0002	4.0040	3.7411	2.540	0.234
10.0008	4.0014	3.7407	2.545	0.235
10.0047	3.9814	3.7367	2.559	0.240
10.0166	3.8945	3.7104	2.482	0.283
10.0228	3.8257	3.6926	2.453	0.316
10.0268	3.7624	3.6834	2.535	0.334
10.0298	3.7042	3.6785	2.679	0.345
10.0322	3.6538	3.6758	2.845	0.350
10.0342	3.6082	3.6741	3.023	0.353
10.0397	3.4626	3.6716	3.769	0.358
10.0431	3.3457	3.6699	4.563	0.359
10.0454	3.2428	3.6681	5.424	0.358
10.0471	3.1520	3.6663	6.337	0.357

TABLE II.2.

Log m = 0.0492

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.4178	3.7838	1.712	0.248
8.8451	4.3955	3.7852	1.763	0.243
9.4152	4.3210	3.7869	1.927	0.229
9.5024	4.2926	3.7862	1.983	0.225
9.5570	4.2691	3.7850	2.025	0.222
9.5936	4.2501	3.7838	2.057	0.221
9.6184	4.2350	3.7828	2.084	0.220
9.6354	4.2227	3.7826	2.114	0.218
9.6465	4.2116	3.7835	2.156	0.215
9.6632	4.1951	3.7838	2.207	0.212
9.7414	4.0974	3.7779	2.441	0.204
9.7948	3.9706	3.7625	2.695	0.208
9.8176	3.8743	3.7375	2.694	0.241
9.8277	3.8140	3.7101	2.488	0.295
9.8333	3.7586	3.6950	2.449	0.327
9.8373	3.7036	3.6874	2.530	0.343
9.8404	3.6539	3.6835	2.657	0.351
9.8430	3.6099	3.6811	2.799	0.356
9.8471	3.5314	3.6783	3.113	0.361
9.8525	3.4069	3.6755	3.754	0.365
9.8562	3.3015	3.6736	4.444	0.365
9.8589	3.2044	3.6715	5.210	0.365
9.8609	3.1201	3.6695	5.996	0.364
9.8624	3.0429	3.6675	6.828	0.363
9.8636	2.9713	3.6655	7.707	0.361
9.8646	2.9047	3.6636	8.632	0.360
9.8654	2.8418	3.6615	9.608	0.358

TABLE II.3.

Log m = 0.0969

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3540	3.8110	1.890	0.224
8.6990	4.3295	3.8116	1.946	0.220
9.2687	4.2456	3.8089	2.110	0.208
9.4215	4.1785	3.8012	2.200	0.205
9.4616	4.1526	3.7969	2.223	0.206
9.4889	4.1321	3.7934	2.239	0.206
9.5075	4.1169	3.7915	2.260	0.206
9.5181	4.1067	3.7923	2.302	0.204
9.5232	4.1007	3.7952	2.361	0.199
9.5256	4.0949	3.7989	2.430	0.194
9.5269	4.0874	3.8017	2.496	0.190
9.5278	4.0803	3.8021	2.532	0.189
9.5289	4.0758	3.8018	2.544	0.188
9.5371	4.0611	3.8002	2.576	0.188
9.5850	3.9754	3.7897	2.761	0.186
9.6240	3.8520	3.7665	2.947	0.199
9.6405	3.7791	3.7302	2.679	0.261
9.6480	3.7413	3.7068	2.450	0.315
9.6530	3.6928	3.6956	2.450	0.340
9.6568	3.6458	3.6902	2.537	0.353
9.6599	3.6027	3.6872	2.656	0.359
9.6627	3.5626	3.6850	2.788	0.363
9.6670	3.4928	3.6823	3.063	0.368
9.6732	3.3788	3.6793	3.625	0.371
9.6775	3.2812	3.6772	4.222	0.371
9.6806	3.1953	3.6750	4.841	0.371
9.6830	3.1171	3.6730	5.495	0.370
9.6849	3.0451	3.6710	6.186	0.369
9.6864	2.9768	3.6690	6.925	0.368
9.6877	2.9138	3.6670	7.693	0.367

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TABLE II.4.

Log m = 0.1492

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3124	3.8450	2.020	0.209
8.6990	4.2728	3.8438	2.096	0.203
9.0120	4.2194	3.8393	2.188	0.198
9.1558	4.1675	3.8328	2.267	0.194
9.2411	4.1197	3.8252	2.327	0.192
9.2967	4.0772	3.8171	2.369	0.192
9.3346	4.0427	3.8098	2.393	0.193
9.3604	4.0163	3.8040	2.407	0.194
9.3778	3.9981	3.8009	2.428	0.194
9.3874	3.9887	3.8015	2.469	0.192
9.3917	3.9859	3.8050	2.525	0.188
9.3936	3.9844	3.8104	2.601	0.183
9.3946	3.9865	3.8164	2.670	0.179
9.3951	3.9858	3.8211	2.732	0.176
9.3953	3.9846	3.8237	2.776	0.174
9.3955	3.9834	3.8240	2.793	0.173
9.3958	3.9746	3.8220	2.812	0.173
9.3976	3.9506	3.8185	2.865	0.172
9.4077	3.9124	3.8122	2.952	0.172
9.4388	3.7835	3.7836	3.155	0.181
9.4559	3.7073	3.7365	2.777	0.255
9.4629	3.6912	3.7126	2.466	0.315
9.4681	3.6523	3.7008	2.430	0.344
9.4758	3.5695	3.6919	2.604	0.364
9.4949	3.3003	3.6820	3.778	0.377
9.4991	3.2200	3.6799	4.276	0.378
9.5048	3.0783	3.6758	5.345	0.376

TABLE II.5.

Log m = 0.1987

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3220	3.8865	2.006	0.209
8.6241	4.2659	3.8820	2.098	0.203
8.9027	4.1973	3.8732	2.205	0.196
9.0358	4.1281	3.8619	2.308	0.191
9.1153	4.0621	3.8493	2.401	0.186
9.1672	4.0043	3.8370	2.473	0.184
9.2023	3.9584	3.8264	2.518	0.184
9.2262	3.9253	3.8188	2.547	0.184
9.2416	3.9044	3.8150	2.578	0.184
9.2499	3.8969	3.8161	2.618	0.182
9.2537	3.8993	3.8208	2.666	0.179
9.2555	3.9059	3.8275	2.718	0.176
9.2564	3.9157	3.8354	2.766	0.173
9.2568	3.9251	3.8430	2.807	0.171
9.2570	3.9318	3.8486	2.840	0.170
9.2571	3.9355	3.8513	2.858	0.169
9.2572	3.9370	3.8515	2.862	0.170
9.2574	3.9330	3.8488	2.868	0.170
9.2585	3.8897	3.8412	2.968	0.167
9.2674	3.8172	3.8274	3.151	0.165
9.2824	3.6705	3.7872	3.376	0.176
9.2910	3.6200	3.7375	2.873	0.256
9.2966	3.6084	3.7106	2.484	0.330
9.3006	3.5591	3.7005	2.506	0.357
9.3097	3.4234	3.6909	2.909	0.377
9.3141	3.3566	3.6882	3.190	0.380
9.3204	3.2609	3.6852	3.681	0.382
9.3250	3.1825	3.6828	4.157	0.382
9.3286	3.1103	3.6806	4.648	0.382
9.3314	3.0450	3.6785	5.141	0.381

TABLE II.6.

Log m = 0.2989

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3458	3.9668	1.907	0.218
5.8451	4.3458	3.9668	1.907	0.218
6.7993	4.3443	3.9666	1.909	0.218
7.7084	4.3319	3.9654	1.926	0.216
8.4263	4.2627	3.9573	2.019	0.209
8.6608	4.1818	3.9453	2.128	0.202
8.7764	4.1026	3.9311	2.234	0.196
8.8469	4.0284	3.9166	2.332	0.191
8.8930	3.9594	3.9015	2.423	0.188
8.9239	3.9004	3.8880	2.501	0.185
8.9449	3.8562	3.8779	2.564	0.182
8.9589	3.8315	3.8734	2.612	0.181
8.9660	3.8309	3.8763	2.639	0.180
8.9694	3.8447	3.8837	2.654	0.179
8.9710	3.8651	3.8934	2.666	0.178
8.9717	3.8857	3.9036	2.681	0.178
8.9721	3.9020	3.9129	2.703	0.177
8.9723	3.9127	3.9197	2.727	0.176
8.9724	3.9186	3.9233	2.744	0.175
8.9725	3.9216	3.9242	2.748	0.175
8.9725	3.9234	3.9236	2.745	0.176
8.9729	3.8890	3.9130	2.805	0.175
8.9745	3.8333	3.9038	2.932	0.171
8.9843	3.6535	3.8606	3.357	0.164
8.9894	3.4654	3.8025	3.741	0.166
8.9932	3.4121	3.7339	2.976	0.266
8.9958	3.3394	3.7035	2.585	0.359
8.9983	3.1764	3.6928	2.961	0.379
9.0001	3.0654	3.6880	3.374	0.383
9.0025	2.9217	3.6820	4.105	0.384
9.0042	2.8284	3.6780	4.738	0.384
9.0056	2.7544	3.6749	5.337	0.384
9.0067	2.6889	3.6721	5.928	0.384
9.0077	2.6357	3.6697	6.474	0.383

TABLE II.7.

Log m = 0.3997

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3505	4.0396	1.816	0.227
5.8451	4.3503	4.0396	1.816	0.227
6.7993	4.3472	4.0393	1.820	0.227
7.7084	4.3220	4.0367	1.851	0.224
8.2493	4.2358	4.0262	1.957	0.215
8.4281	4.1529	4.0137	2.060	0.208
8.5241	4.0734	3.9998	2.158	0.202
8.5829	3.9995	3.9853	2.250	0.197
8.6218	3.9339	3.9715	2.332	0.194
8.6483	3.8787	3.9595	2.403	0.191
8.6666	3.8397	3.9514	2.460	0.188
8.6777	3.8236	3.9497	2.499	0.187
8.6833	3.8300	3.9546	2.518	0.186
8.6859	3.8477	3.9628	2.529	0.186
8.6871	3.8695	3.9726	2.539	0.185
8.6878	3.8905	3.9827	2.552	0.184
8.6881	3.9058	3.9910	2.572	0.183
8.6882	3.9149	3.9964	2.591	0.183
8.6883	3.9197	3.9988	2.602	0.182
8.6883	3.9220	3.9993	2.604	0.182
8.6884	3.9226	3.9967	2.598	0.183
8.6887	3.8832	3.9870	2.669	0.181
8.6914	3.8031	3.9732	2.842	0.176
8.7013	3.4082	3.8648	3.693	0.166
8.7036	3.1673	3.7733	4.039	0.183
8.7047	3.1709	3.7297	3.256	0.269
8.7064	3.0068	3.6969	3.093	0.367
8.7081	2.7968	3.6852	3.843	0.380
8.7101	2.5717	3.6741	5.232	0.382

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TABLE II.8.

Log m = 0.4997

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3460	4.1085	1.736	0.236
5.8451	4.3455	4.1084	1.736	0.236
6.7993	4.3397	4.1079	1.743	0.235
7.7084	4.2905	4.1032	1.799	0.230
8.0542	4.2039	4.0926	1.900	0.221
8.1962	4.1218	4.0805	1.996	0.214
8.2754	4.0451	4.0675	2.086	0.208
8.3252	3.9749	4.0542	2.169	0.204
8.3585	3.9138	4.0419	2.243	0.200
8.3812	3.8652	4.0321	2.305	0.197
8.3966	3.8345	4.0267	2.354	0.195
8.4056	3.8266	4.0275	2.385	0.193
8.4099	3.8370	4.0332	2.400	0.193
8.4120	3.8573	4.0419	2.408	0.192
8.4130	3.8799	4.0515	2.415	0.192
8.4134	3.8994	4.0605	2.428	0.191
8.4137	3.9127	4.0676	2.447	0.190
8.4138	3.9202	4.0717	2.462	0.190
8.4139	3.9236	4.0731	2.469	0.190
8.4139	3.9252	4.0729	2.469	0.190
8.4141	3.9195	4.0676	2.469	0.191
8.4148	3.8499	4.0560	2.604	0.186
8.4179	3.7729	4.0414	2.761	0.182
8.4253	3.4155	3.9468	3.513	0.171
8.4271	3.1466	3.8594	4.125	0.170
8.4279	2.9239	3.7735	4.557	0.181
8.4287	2.8943	3.7183	3.723	0.283
8.4295	2.6694	3.6888	4.009	0.364

TABLE II.9.

Log m = 0.5999

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3366	4.1750	1.664	0.245
5.8451	4.3355	4.1749	1.665	0.245
6.7993	4.3253	4.1741	1.677	0.243
7.5848	4.2580	4.1679	1.750	0.236
7.8389	4.1758	4.1583	1.841	0.228
7.9577	4.0959	4.1471	1.931	0.220
8.0279	4.0204	4.1349	2.018	0.214
8.0722	3.9518	4.1228	2.097	0.209
8.1014	3.8935	4.1119	2.167	0.206
8.1211	3.8487	4.1036	2.226	0.203
8.1344	3.8240	4.1003	2.270	0.201
8.1415	3.8236	4.1029	2.293	0.199
8.1449	3.8391	4.1097	2.303	0.199
8.1466	3.8611	4.1183	2.309	0.199
8.1474	3.8837	4.1275	2.316	0.198
8.1478	3.9013	4.1354	2.330	0.197
8.1480	3.9128	4.1411	2.346	0.197
8.1481	3.9184	4.1438	2.357	0.197
8.1482	3.9206	4.1444	2.362	0.197
8.1482	3.9212	4.1441	2.364	0.197
8.1484	3.9112	4.1379	2.371	0.197
8.1491	3.8274	4.1244	2.526	0.191
8.1534	3.6937	4.0972	2.788	0.184
8.1583	3.2187	3.9647	3.800	0.174
8.1592	2.9779	3.8848	4.388	0.174
8.1597	2.7296	3.7952	5.052	0.179

TABLE II.10.

Log m = 0.6998

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3276	4.2393	1.598	0.253
5.8451	4.3256	4.2392	1.600	0.253
6.7993	4.3074	4.2377	1.619	0.251
7.4358	4.2270	4.2311	1.705	0.241
7.6377	4.1426	4.2218	1.797	0.233
7.7363	4.0640	4.2111	1.883	0.225
7.7958	3.9911	4.1997	1.965	0.219
7.8346	3.9254	4.1886	2.040	0.214
7.8604	3.8705	4.1788	2.107	0.210
7.8783	3.8300	4.1720	2.162	0.208
7.8898	3.8114	4.1703	2.201	0.205
7.8955	3.8162	4.1742	2.220	0.205
7.8984	3.8340	4.1812	2.228	0.204
7.8998	3.8567	4.1897	2.234	0.204
7.9005	3.8782	4.1981	2.242	0.204
7.9008	3.8945	4.2052	2.255	0.203
7.9010	3.9042	4.2098	2.270	0.202
7.9011	3.9086	4.2118	2.281	0.202
7.9012	3.9102	4.2122	2.286	0.202
7.9012	3.9103	4.2115	2.289	0.202
7.9014	3.8812	4.2009	2.323	0.202
7.9026	3.7686	4.1833	2.532	0.194
7.9072	3.4834	4.1167	3.093	0.182
7.9095	3.0206	3.9823	4.208	0.175
7.9101	2.7178	3.8820	5.108	0.177

TABLE II.11.

Log m = 0.7993

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3147	4.3003	1.543	0.261
5.8451	4.3112	4.3001	1.546	0.261
6.7993	4.2800	4.2982	1.578	0.257
7.2810	4.1922	4.2910	1.670	0.246
7.4429	4.1095	4.2818	1.758	0.237
7.5297	4.0312	4.2714	1.844	0.230
7.5826	3.9588	4.2604	1.925	0.224
7.6170	3.8948	4.2500	1.999	0.218
7.6401	3.8418	4.2410	2.065	0.214
7.6559	3.8050	4.2352	2.119	0.211
7.6659	3.7915	4.2348	2.155	0.209
7.6708	3.7996	4.2392	2.172	0.209
7.6732	3.8188	4.2463	2.179	0.208
7.6745	3.8415	4.2545	2.185	0.208
7.6751	3.8623	4.2623	2.194	0.208
7.6754	3.8765	4.2681	2.209	0.207
7.6756	3.8857	4.2725	2.224	0.206
7.6756	3.8897	4.2743	2.235	0.206
7.6757	3.8912	4.2747	2.241	0.206
7.6757	3.8908	4.2736	2.246	0.206
7.6760	3.8333	4.2580	2.329	0.204
7.6774	3.6726	4.2291	2.625	0.194
7.6803	3.3272	4.1440	3.351	0.182
7.6816	2.8757	4.0137	4.607	0.176
7.6820	2.5668	3.9161	5.737	0.176

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TABLE II.12.

Log m = 0.8998

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.3012	4.3584	1.493	0.269
5.8451	4.2952	4.3581	1.499	0.268
6.7931	4.2431	4.3552	1.552	0.261
7.1265	4.1555	4.3477	1.643	0.250
7.2605	4.0731	4.3385	1.732	0.241
7.3359	3.9953	4.3282	1.818	0.233
7.3832	3.9230	4.3173	1.900	0.227
7.4149	3.8583	4.3068	1.978	0.221
7.4365	3.8063	4.2984	2.046	0.217
7.4512	3.7716	4.2935	2.102	0.214
7.4596	3.7628	4.2940	2.136	0.212
7.4638	3.7734	4.2988	2.152	0.211
7.4659	3.7932	4.3058	2.160	0.211
7.4669	3.8153	4.3131	2.167	0.211
7.4675	3.8358	4.3209	2.178	0.210
7.4677	3.8503	4.3266	2.192	0.209
7.4679	3.8587	4.3303	2.208	0.209
7.4680	3.8628	4.3322	2.221	0.209
7.4680	3.8647	4.3332	2.232	0.208
7.4680	3.8653	4.3335	2.238	0.208
7.4682	3.8503	4.3252	2.252	0.209
7.4688	3.6895	4.2938	2.541	0.199
7.4703	3.4094	4.2308	3.126	0.187
7.4713	3.0932	4.1473	3.975	0.179
7.4719	2.7643	4.0531	5.129	0.175
7.4722	2.4856	3.9698	6.398	0.174

TABLE II.13.

Log m = 1.0000

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.2880	4.4125	1.449	0.276
5.8451	4.2789	4.4121	1.458	0.275
6.7000	4.2139	4.4086	1.525	0.265
6.9668	4.1268	4.4011	1.617	0.254
7.0903	4.0421	4.3916	1.710	0.244
7.1608	3.9617	4.3809	1.802	0.236
7.2056	3.8871	4.3697	1.891	0.228
7.2354	3.8211	4.3592	1.974	0.222
7.2558	3.7676	4.3504	2.049	0.218
7.2697	3.7337	4.3456	2.108	0.214
7.2774	3.7261	4.3463	2.144	0.213
7.2812	3.7371	4.3510	2.162	0.212
7.2832	3.7571	4.3577	2.171	0.211
7.2842	3.7791	4.3651	2.180	0.211
7.2847	3.7989	4.3719	2.192	0.211
7.2850	3.8135	4.3775	2.208	0.210
7.2851	3.8225	4.3813	2.226	0.209
7.2852	3.8274	4.3836	2.241	0.209
7.2852	3.8301	4.3850	2.257	0.208
7.2853	3.8316	4.3856	2.273	0.208
7.2855	3.7750	4.3684	2.366	0.207
7.2864	3.4605	4.3021	3.030	0.190
7.2872	3.1468	4.2239	3.895	0.180
7.2878	2.7832	4.1253	5.277	0.174

TABLE II.14.

Log m = 1.0997

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.2759	4.4623	1.411	0.283
5.8451	4.2623	4.4618	1.425	0.280
6.6117	4.1848	4.4573	1.506	0.269
6.8418	4.0950	4.4494	1.605	0.256
6.9506	4.0086	4.4393	1.703	0.245
7.0147	3.9261	4.4279	1.802	0.236
7.0558	3.8495	4.4163	1.898	0.229
7.0835	3.7818	4.4051	1.990	0.222
7.1027	3.7272	4.3962	2.072	0.217
7.1156	3.6944	4.3916	2.137	0.214
7.1226	3.6883	4.3924	2.176	0.212
7.1261	3.6995	4.3969	2.195	0.211
7.1280	3.7192	4.4033	2.207	0.210
7.1289	3.7408	4.4103	2.217	0.210
7.1294	3.7608	4.4169	2.230	0.210
7.1297	3.7759	4.4223	2.248	0.209
7.1298	3.7859	4.4264	2.268	0.208
7.1299	3.7920	4.4290	2.286	0.208
7.1299	3.7958	4.4307	2.306	0.207
7.1299	3.7985	4.4320	2.327	0.206
7.1301	3.7636	4.4210	2.414	0.206
7.1306	3.5163	4.3672	2.974	0.192
7.1312	3.1770	4.2860	3.992	0.180
7.1318	2.7226	4.1666	6.048	0.170

TABLE II.15.

Log m = 1.1998

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.2652	4.5082	1.379	0.289
5.8451	4.2462	4.5075	1.399	0.285
6.5138	4.1605	4.5021	1.491	0.271
6.7142	4.0703	4.4934	1.593	0.258
6.8165	3.9814	4.4827	1.700	0.247
6.8778	3.8964	4.4705	1.807	0.237
6.9181	3.8157	4.4579	1.916	0.228
6.9455	3.7442	4.4458	2.020	0.221
6.9644	3.6869	4.4361	2.115	0.215
6.9770	3.6529	4.4311	2.190	0.211
6.9837	3.6464	4.4316	2.233	0.209
6.9872	3.6574	4.4358	2.256	0.208
6.9890	3.6768	4.4419	2.269	0.208
6.9899	3.6992	4.4487	2.279	0.208
6.9904	3.7202	4.4553	2.292	0.207
6.9907	3.7368	4.4609	2.310	0.206
6.9909	3.7489	4.4653	2.332	0.206
6.9909	3.7568	4.4684	2.354	0.205
6.9910	3.7624	4.4711	2.382	0.204
6.9910	3.7680	4.4737	2.417	0.203
6.9912	3.7196	4.4608	2.586	0.201
6.9918	3.3510	4.3779	3.667	0.182
6.9924	2.8014	4.2398	6.333	0.165

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TABLE II.16.

Log m = 1.2999

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.2557	4.5498	1.352	0.294
5.8451	4.2303	4.5488	1.380	0.289
6.4366	4.1361	4.5423	1.485	0.273
6.6149	4.0432	4.5326	1.596	0.258
6.7107	3.9497	4.5205	1.716	0.245
6.7692	3.8592	4.5068	1.839	0.234
6.8077	3.7735	4.4932	1.968	0.224
6.8342	3.6969	4.4797	2.091	0.216
6.8527	3.6356	4.4688	2.205	0.210
6.8649	3.5996	4.4630	2.294	0.205
6.8713	3.5917	4.4630	2.346	0.203
6.8747	3.6013	4.4667	2.374	0.202
6.8765	3.6202	4.4725	2.390	0.202
6.8774	3.6432	4.4792	2.400	0.201
6.8779	3.6649	4.4857	2.414	0.201
6.8782	3.6834	4.4915	2.433	0.200
6.8784	3.6977	4.4963	2.455	0.200
6.8785	3.7082	4.5000	2.480	0.199
6.8785	3.7170	4.5036	2.518	0.198
6.8785	3.7278	4.5080	2.579	0.196
6.8787	3.6960	4.5008	2.784	0.193
6.8792	3.3568	4.4236	3.986	0.175
6.8796	2.9413	4.3237	6.391	0.158

TABLE II.17.

Log m = 1.3998

Log Age	Log g	Log Teff	Alpha	Beta
0.0000	4.2474	4.5870	1.330	0.299
5.8451	4.2142	4.5857	1.368	0.292
6.3580	4.1178	4.5785	1.482	0.274
6.5291	4.0199	4.5675	1.607	0.257
6.6199	3.9217	4.5537	1.742	0.243
6.6771	3.8229	4.5380	1.890	0.229
6.7148	3.7295	4.5217	2.041	0.218
6.7410	3.6465	4.5067	2.192	0.209
6.7593	3.5786	4.4940	2.333	0.202
6.7714	3.5374	4.4866	2.445	0.197
6.7779	3.5251	4.4854	2.514	0.194
6.7813	3.5321	4.4884	2.551	0.193
6.7832	3.5491	4.4935	2.573	0.193
6.7842	3.5711	4.4998	2.587	0.192
6.7848	3.5935	4.5063	2.601	0.192
6.7851	3.6135	4.5122	2.620	0.191
6.7852	3.6302	4.5173	2.643	0.191
6.7853	3.6430	4.5214	2.670	0.190
6.7854	3.6540	4.5249	2.702	0.189
6.7854	3.6611	4.5282	2.751	0.188
6.7855	3.6801	4.5346	2.894	0.186
6.7859	3.4403	4.4814	3.944	0.170
6.7862	3.1411	4.4115	5.622	0.155

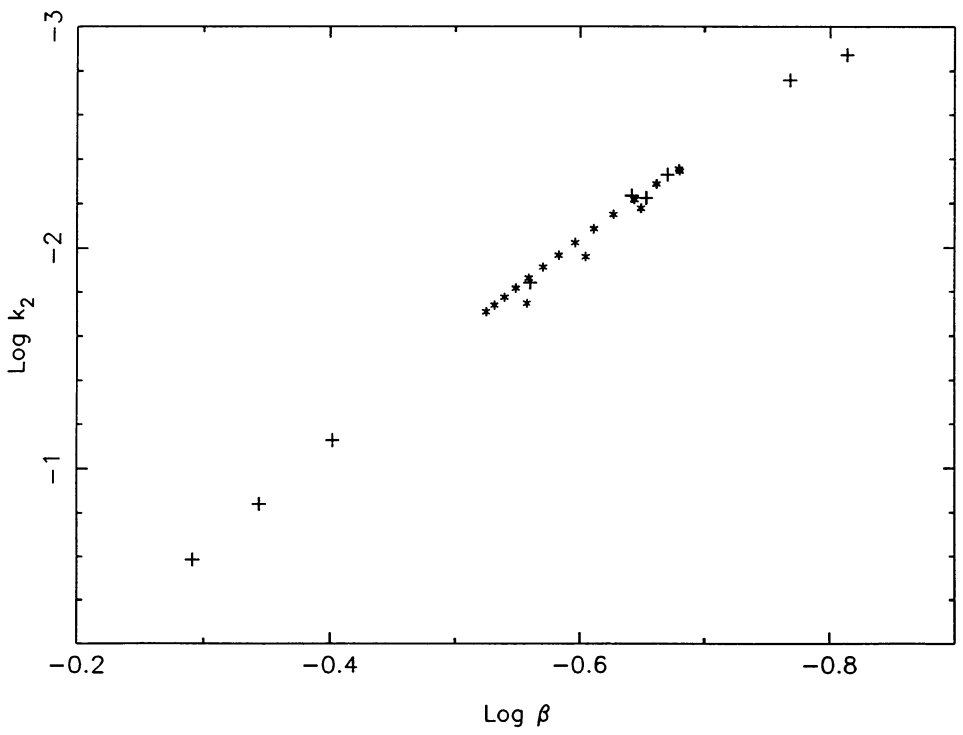

FIGURE 1. — The log k_2 – log β relation. Results by Motz (1952) are plotted as crosses (+) and our ZAMS models as asterisks (*). The turnover point shown in figure 2 can also be noticed here through a slightly different slope.

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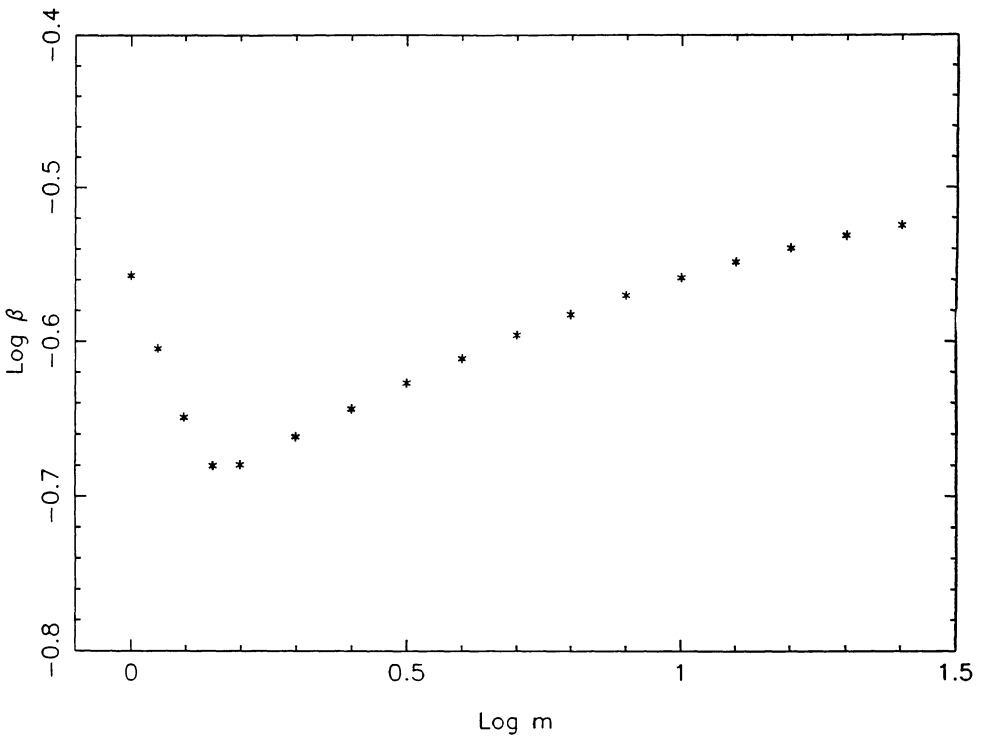

FIGURE 2. — The log β – log m relation. Asterisks (*) denote computed values for ZAMS models from table I.

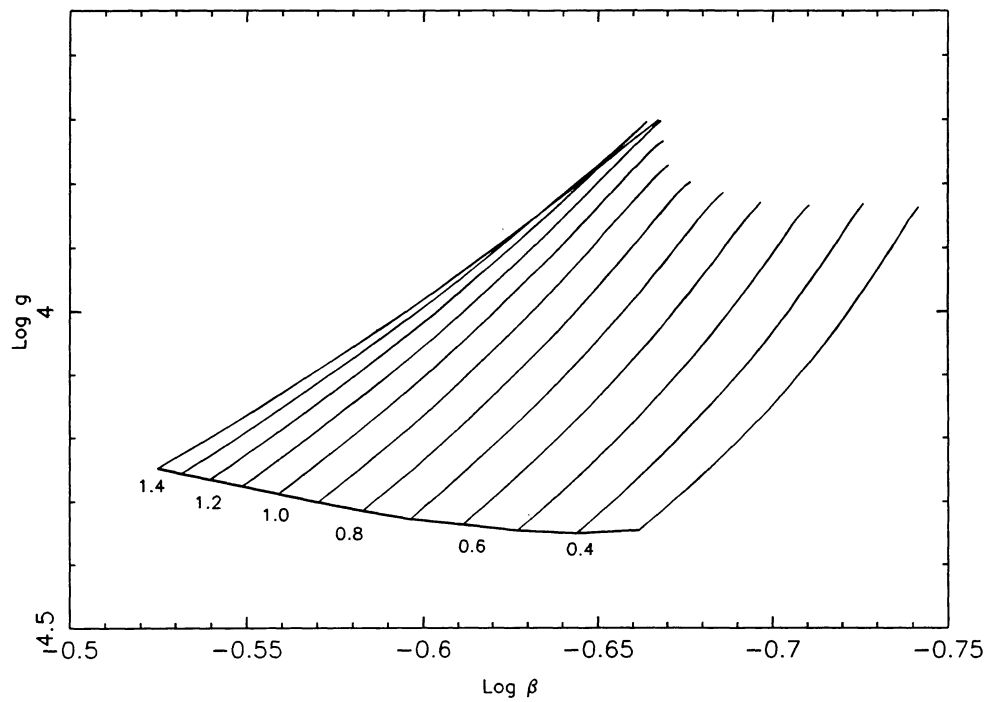

FIGURE 3. — Evolved models during the main sequence, from the ZAMS up to the TAMS in the log g – log β plane. Each other evolutionary track is labelled with its corresponding logarithm of the mass in solar units. A step of 0.1 in log m was adopted and low-mass models (log m < 0.3) are not plotted for the sake of clarity.

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